Uncertainties in Assessing the Corrosion Wastage and its Effect on Ship Structure Scantlings*

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Summary

The paper discusses the uncertainty of corrosion wastage and its effect on steel plates’ thickness. The composition of distribution laws of the constituent variables is used for describing time-variant probabilistic distributions of residual plate thickness when the as-built thickness and the corrosion wastage obey any probabilistic distribution. The results compare satisfactorily with the Monte Carlo simulation method. Numerical examples are given for the inner bottom, bottom shell, side shell and deck plating of a 25,000 DWT bulk carrier for a case in which the as-built plate thickness obeys the normal distribution and that the corrosion wastage follows either the Weibull or normal (including truncated normal) distribution. The probability that the residual thickness of a structural member meets given Renewal Criteria requirements is calculated. A conclusion is made that any of these three distribution laws are suitable for calculating the probability that a given plate thickness will meet specified Renewal Criteria requirements.

Key Words

Corrosion wastage; probabilistic methods

Introduction

The plate thickness of a structural member at any ship’s age, T, can be presented as:

\[ t_T = t_0 - \delta_T \]  

where:

- \( t_T \) = residual plate thickness at time \( T \)
- \( t_0 \) = as-built plate thickness
- \( \delta_T \) = corrosion wastage at time \( T \)

Often, the focus of studies on the strength of aging ships has been on the mean and variance/standard deviation of corrosion wastage. This can be done using the formulae in the theory of probabilities. However, there is also a need to know the effect of the type of the probabilistic distribution on the geometric properties and stress distribution. The paper aims at developing a general analytical method for obtaining the probabilistic distribution of the residual thickness, \( t_T \), which can be used in time-variant reliability calculations of ship structures.

As-built Thickness

The as-built plate thickness differs from the nominal values mostly because of manufacturing tolerance. There are only a few data published for the probabilistic distributions of as-built thickness. Vasilev, Gluskina, (1973) suggested that the initial thickness, \( t_0 \), follows the normal distribution

\[ f_{t_0}(t_0) = \frac{1}{\sigma_{t_0} \sqrt{2\pi}} \exp \left( -\frac{1}{2} \left( \frac{t_0 - \overline{t_0}}{\sigma_{t_0}} \right)^2 \right) \]  

where:

- \( f_{t_0}(t_0) \) = probability density function (p.d.f.) of \( t_0 \)
- \( \overline{t_0} \) = mean value of the initial thickness \( t_0 \)
- \( \sigma_{t_0} \) = standard deviation of the initial thickness

An example of the inner bottom plating of a 25,000 DWT bulk carrier is given in Fig. 1.

Corrosion Wastage

There are some ongoing efforts (Wang, 2003, Paik, 2003) in the collection of corrosion wastage measurement data from commercial ships, and the development of some functions to describe the observed data. The most often used assumptions for corrosion wastage are normal (including truncated normal), log-normal and Weibull distributions.

*The views expressed in this paper are those of the authors, and not necessarily those of the American Bureau of Shipping.
Maximadji et al (1982) suggested that the corrosion wastage obeys the normal distribution, based on continuous surveying and gauging of corrosion wastage in tankers and dry cargo ships.

Paik et al. (2003) performed probabilistic analysis of collected corrosion wastage data for bulk carriers. The annualized corrosion rate, $\bar{C}$, was assumed as a constant throughout the ship’s life and obeying the Weibull distribution. Then, the corrosion wastage, $\delta_T$, is presented in the following way:

$$\delta_T = T_e \cdot C$$

(3)

where:
- $T_e = T - T_c - T_t$ time after breakdown of coating
- $T$ = given time
- $T_c$ = coating’s life in years
- $T_t$ = duration of transition period in years after the coatings breaks down (in this paper, $T_c$ and $T_t$ are treated as deterministic magnitudes)
- $C$ = annualized corrosion rate. Its p.d.f. and cumulative distribution function (c.d.f.) are given in Eq. (4) where $f_C(C)$ is the p.d.f. and $F_C(C)$ is the c.d.f. of $C$.

$$f_C(C) = \frac{\lambda_c}{\alpha_c} C^{\alpha_c - 1} \exp \left( - \frac{C}{\alpha_c} \right)$$

$$F_C(C) = 1 - \exp \left( - \frac{C}{\alpha_c} \right)$$

(4)

$\lambda_c, \alpha_c$ = shape and scale parameters of $C$

The mean and variance of $C$ can be calculated by the following equations (Ochi, 1989):

$$\overline{C} = \alpha_c \Gamma \left( 1 + \frac{1}{\lambda_c} \right)$$

$$D_C = \alpha_c^2 \left( \Gamma \left( 1 + \frac{2}{\lambda_c} \right) - \left[ \Gamma \left( 1 + \frac{1}{\lambda_c} \right) \right]^2 \right)$$

(5)

Using $\overline{C}$ and $D_C$, given by Paik et al (2003), $\lambda_c$ and $\alpha_c$ were found by solving systems of two equations as shown in Eq. (5) for each structural component.

The p.d.f. of $C$ for the inner bottom plating of bulk carriers (Paik et al, 2003) is given in Fig. 2.

The corrosion wastage of the inner bottom plating of a 25,000 DWT bulk carrier is shown in Fig. 3. As it can be observed in that figure, the spread of corrosion wastage increases substantially with the ship’s aging.

**Residual Thickness**

Knowing the distribution laws of the as-built thickness, $t_0$, and the corrosion wastage, $\delta_T$, the probabilistic distribution of the residual thickness, $t_T$, at any ship’s age is calculated using the composition of the distribution laws of the constituent variables (Livshits, Pugachev, 1963 and Suhir, 1997). Because the thickness cannot be negative, the integration starts from zero.

Fig. 1 P.d.f. of the as-built thickness, $t_0$, for the inner bottom plating of a 25,000 DWT bulk carrier.

Fig. 2 P.d.f. of the annualized corrosion wastage $C$ for the inner bottom plating (Paik et al, 2003)

Fig. 3 P.d.f. of the corrosion wastage, $\delta_T$, for the inner bottom plating of a 25,000 DWT bulk carrier.
Eq. (1) can be rewritten in the following way:

\[ t_r = t_0 - \delta_r = t_0 - T_e C \]  
(6)

Then, the mean value and the variance of \( t_r \) are

\[
\overline{t_r} = t_0 - \overline{\delta_r} = t_0 - T_e \overline{C} \\
D_{t_r} = D_{t_0} + D_{\delta_r} = D_{t_0} + T_e^2 D_C
\]  
(7)

where:

- \( \overline{t_r} \) = mean value of \( t_r \)
- \( D_{t_r} \) = variance of \( t_r \)
- \( t_0 \) = mean value of \( t_0 \)
- \( D_{t_0} \) = variance of \( t_0 \)
- \( \overline{\delta_r} = T_e \overline{C} \) = mean value of \( \overline{\delta_r} \)
- \( D_{\delta_r} = T_e^2 D_C \) = variance of \( \overline{\delta_r} \)
- \( C = \) mean value of \( C \)
- \( D_C = \) variance of \( C \)

For a given instant of time, \( T_e \) is a constant. Hence, \( \overline{\delta_r} \) obeys the Weibull distribution with scale and shape parameters \( \alpha_{\delta_r} \) and \( \lambda_{\delta_r} \):

\[
\overline{\delta_r} = \alpha_{\delta_r} \Gamma\left(1 + \frac{1}{\lambda_{\delta_r}}\right) \\
D_{\delta_r} = \alpha_{\delta_r}^2 \left[\Gamma\left(1 + \frac{2}{\lambda_{\delta_r}}\right) - \left[\Gamma\left(1 + \frac{1}{\lambda_{\delta_r}}\right)\right]^2\right]
\]  
(10)

where:

- \( \overline{\delta_r} \) = mean value of \( \delta_r \)
- \( D_{\delta_r} \) = variance of \( \delta_r \)

The probability density function of \( t_r \) is:

\[
f_{t_r}(t_r) = f_{\delta_r}(\delta_r) f_{t_0}(t_r + \delta_r) d(\delta_r)
\]  
(11)

where:

- \( f_{t_r}(t_r) \) = probability density function of \( t_r \)
- \( f_{\delta_r}(\delta_r) \) = probability density function of \( \delta_r \)

\[
f_{\delta_r}(\delta_r) = \frac{\lambda_{\delta_r}}{\alpha_{\delta_r}} \delta_r^{\lambda_{\delta_r}-1} \exp\left[-\left(\frac{\delta_r}{\alpha_{\delta_r}}\right)^{\lambda_{\delta_r}}\right]
\]  
(12)

Figs. 5 and 6 present the p.d.f. of the residual thickness, \( t_r \), assuming it obeys the truncated normal distribution. The probability of plate thickness exceeding the Renewal Criteria (ABS Rules, 2002) is presented as a means of assessing the structural integrity.
The area below the p.d.f., for any ship’s age, that is located to the left of the minimum permissible thickness determines the probability of failing to meet the Renewal Criteria for a given ship’s age. All curves in Fig. 7 are built for a case where the numerical characteristics of the three distribution laws are the same. Therefore, the effect of the distribution type is clearly observed. The difference in the predicted probabilities of meeting the Renewal Criteria requirements calculated with the three distribution laws is negligible, which can be seen in the following examples as well.

**Bottom Shell Plating**

The p.d.f. of \( C \) for the bottom shell plating of bulk carriers with data of Paik et al (2003), who assumed 7.5 years of coating longevity, is given in Fig. 8.

One can see that, in this case, the p.d.f. of \( C \) is close to exponential distribution (the thickness of the bottom shell plating is the same as that of the inner bottom). Therefore, its probability density function is the same as that in Fig. 1. The results of the calculations for the corrosion wastage, \( \delta \), of the bottom shell plating of a 25,000 DWT bulk carrier are shown in Fig. 9.
The p.d.f. of the residual thickness, $t_r$, for the bottom shell plating of the same bulk carrier was calculated again by Eq. (11). The results are shown in Fig. 10, together with the permissible minimum thickness of the plating according to the Renewal Criteria (ABS Rules, 2002). One can again observe that, with ship’s aging, the p.d.f. of the residual thickness, $t_r$, deviates more and more from the normal distribution.

The p.d.f. of $t_r$ for the bottom shell plating of a 25,000 DWT bulk carrier when it obeys the Weibull distribution is shown in Fig. 11 (the lower truncation is at $0.5t_0$, the upper truncation is at $1.05t_0$). The normal distribution of $t_r$ of the bottom shell plating (with the same mean value and standard deviation as the truncated normal and Weibull distribution) is shown in Fig. 12.

The mean values and variances of the normal and truncated normal distributions of $t_r$ of the bottom shell plating for the same bulk carrier were calculated by Eq. (7). The truncated normal distribution of $t_r$ is shown in Fig. 11. It confirms the similarity of the results when the three distribution laws are used.

Deck Plating

The p.d.f. of the residual thickness, $t_r$, for the bottom shell plating of the same bulk carrier was calculated again by Eq. (11). The results are shown in Fig. 10, together with the permissible minimum thickness of the plating according to the Renewal Criteria (ABS Rules, 2002). One can again observe that, with ship’s aging, the p.d.f. of the residual thickness, $t_r$, deviates more and more from the normal distribution.

The p.d.f. of $t_r$ for the bottom shell plating of a 25,000 DWT bulk carrier when it obeys the normal distribution is shown in Fig. 12. The results of the calculations for the probability that the residual bottom shell plating will meet the Renewal Criteria (ABS Rules, 2002) are shown in Fig. 13 where all curves are built with the same numerical characteristics for the three probabilistic distributions.

Deck Plating

The p.d.f. of $C$ for the deck plating of bulk carriers built with data of Paik et al (2003), who assumed 7.5 years coating longevity, is given in Fig. 14 and the p.d.f. of $t_0$ of the deck plating is shown in Fig. 15.
The p.d.f. of the corrosion wastage of the deck plating of the same bulk carrier are shown in Fig. 16. The p.d.f. of the residual thickness, $t_r$, for the deck plating of the same bulk carrier was calculated by Eq. (11) and the results are shown in Fig. 17.

One can again observe in Fig. 17 that with the passing of time, the p.d.f. of the residual thickness, $t_r$, deviates more and more from the normal distribution.

The truncated normal distribution of $t_0$ of the deck plating of the same bulk carrier is shown in Fig. 18. The normal distribution of the deck plating residual thickness with the same mean value and standard deviation as for the truncated normal and Weibull distribution is shown in Fig. 19.

The results of the numerical calculations for the probability that the residual deck plating will meet the Renewal Criteria requirements (ABS Rules, 2002) are shown in Fig. 20. Again, very good agreement between the calculated probabilities of meeting the Renewal Criteria requirements is observed when the three distribution laws are applied.
**Side Plating**

The p.d.f. of $C$ for the side plating of bulk carriers built with data of Paik et al (2003), who assumed 7.5 years of coating longevity, is given in Fig. 21.

The p.d.f. of $t_0$ of the side plating is the same as $t_0$ shown in Fig. 1. The results of the calculations for its corrosion wastage are presented in Fig. 22. The p.d.f of the residual thickness, $t_r$, for the side plating of the same bulk carrier was again calculated by Eq. (11).

![Fig. 19 P.d.f. of the residual deck plating thickness of a 25,000 DWT bulk carrier when it obeys the normal distribution](image1)

![Fig. 20 Probability of meeting the Renewal Criteria requirements (ABS Rules, 2002) for the deck plating of a 25,000 DWT bulk carrier when different distribution laws are used (20% permissible reduction of $t_0$)](image2)

One can observe in Fig. 23 the same trend that, with ship’s aging, the p.d.f. of the residual thickness, $t_r$, deviates more and more from the normal distribution.

The mean values and variances of the Weibull, normal and truncated normal distributions of the residual thickness, $t_r$, of the side plating were again calculated by Eq. (7).
The truncated normal distribution of $t_0$ of the side plating of the same bulk carrier is shown in Fig. 24, while its normal distribution is shown in Fig. 25. The results of the calculations for the probabilities that the residual side plating thickness meets the Renewal Criteria requirements (ABS Rules, 2002) are shown in Fig. 26. In this case, the same good agreement between the results obtained with the three distribution laws can be observed.

Fig. 23 P.d.f. of $t_T$ for the side plating thickness of a 25,000 DWT bulk carrier

Fig. 24 P.d.f. of the residual side plating of a 25,000 DWT bulk carrier when it obeys the truncated normal distribution

Fig. 25 P.d.f. of the side plating thickness of a 25,000 DWT bulk carrier when it obeys the normal distribution

Fig. 26 Probabilities of meeting the Renewal Criteria (ABS Rules, 2002) for the side plating thickness when different distribution laws are used (25% permissible reduction of $t_0$)
Conclusions

An analytical method is used for describing the time-variant probabilistic distributions of the residual plate thickness that changes over the years due to corrosion wastage. It can be used as an alternative to the Monte Carlo simulation method.

Any of the Weibull, normal, and truncated normal distributions can be applied to calculate the probabilities that given plate thickness will meet specific Renewal Criteria requirements at any ship’s age. The normal and truncated normal distributions produce slightly more optimistic results relative to results obtained with Weibull distribution for the corrosion wastage (the difference is around 2-3%).

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References
