A Finite Element Computation for Three Dimensional Problems of Sloshing in LNG Tank

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ABSTRACT
A three-dimensional finite-element method is developed to calculate the impact pressure due to liquid sloshing in LNG tank. The finite-element method presented here is based on potential theory and Hamilton’s principle. As a validation of the developed numerical scheme, the impact sloshing pressure is compared with experiments and previous numerical results. The comparison of the impact pressure due to diagonal motion with the previous two-dimensional impact load shows that three-dimensional effect needs to be considered to estimate correct impact pressure at the upper corners.

INTRODUCTION
Membrane-type LNG carriers transport natural gas in liquid phase near the atmospheric pressure. The temperature of the liquid cargo is at its boiling point, which is about -163°C. To maintain this extremely low temperature, the liquid cargo is separated from the steel hull of the LNG carrier by insulation system that is made of plywood box filled with light insulation material or polyurethane foam. The insulation system should sustain the static and hydrodynamic load from the liquid cargo. The surface of the insulation system is covered by thin metal alloy membrane to prevent the leakage of the liquid cargo.

LNG carriers usually have 4 or 5 tanks. All of the tanks are fully loaded up to 98% of its capacity at the beginning of their transport voyage. During the voyage, small amount of the LNG is boiled off and the LNG tanks are slacked unless it is reliquified. The filling level of the liquid cargo depends on the voyage period and the efficiency of the insulation system. The maximum allowed boil-off rate is 0.15% of the total volume per day. Assuming 20 days of transport voyage, the LNG tank can be slacked to 95% filling level. A more conservative 80% filling level is used for the design filling level for sloshing load. If the LNG carrier encounters heavy weather condition during its voyage with the slacked liquid cargo, large impact pressure on the insulation system can occur due to sloshing motion of the liquid inside the tank.

Recently there has been growing demand and interests for the operations in all-filling level conditions due to the emergence of spot market. The FPSO/FSRU and their shuttle vessels also demand operations in partial loading cases. The sloshing motion at low filling level is known to be quite different from that in the high filling level. At the high filling level, standing waves are observed in the tank. When the filling level is lower than 20% of the tank length or width, progressive waves are observed near the resonant conditions. When the progressive wave breaks on the tank wall, large impact pressure over wide area of tank wall is observed.

The impact pressure on the tank wall due to sloshing is of great concern for the owners, designers and builders of membrane-type LNG carriers, regarding the safety of LNG containment.
system and hull structure. Class societies have been developing analysis procedure to provide reasonable design sloshing load on these structures. Currently, most of the class societies prefer sloshing model test to the numerical simulation. Correct simulation of sloshing impact pressure requires high resolution of local phenomena including but not limited to wave breaking, jet formation, gas entrapping, liquid-gas interaction and fluid-structure interaction, many of which are still in developing stage for the existing academic and commercial numerical tools that are capable of sloshing analysis. Class societies are utilizing numerical tools to evaluate relative measure of sloshing load for different cargo tank and vessel designs.

Many theoretical and numerical methods have been proposed to analyze the flow and pressure pattern in sloshing problem. Faltinsen (1978), Faltinsen & Rognebakke (2000), Faltinsen et al (2000) and Faltinsen & Timokha (2001) used the semi-analytic analysis based on nonlinear modal analysis and perturbation method. The finite-difference method has been one of the most popular numerical method for sloshing problem since the development of SOLA, VOF and SURF schemes (see Chan and Street, 1970 and Hirt et al, 1975). Recent practical application of these schemes can be found in Mikalis (1984), Arai (1994), and Kim (2000). More thorough review on the recent theoretical and numerical work based on the finite-difference scheme can be found in Faltinsen & Rognebakke (2000) and Faltinsen & Timokha (2001). Another popular numerical method is finite-element method. To name a few, Wu et al. (1998) and Kyoung et al. (2003) applied the finite-element method to sloshing problems.

A finite-element code, SLOFE2D, had been developed to analyze the sloshing pressure in two dimensions (Kim et al, 2002). This code is based on stream-function theory and Hamilton’s principle. Since the incompressibility of the liquid is strictly satisfied, the SLOFE2D can capture the impact pressure very accurately. Large impact pressure at the low filling level has been calculated even when the angle between liquid surface and tank wall is not that small, say around 10 degrees. The two dimensional code is useful to find out the critical impact load when the ship is either in head sea or beam sea condition. It has been combined with the seakeeping analysis tools to develop an analysis procedure for critical sloshing load on the insulation system and hull for a given ship dimensions, geometry and period of service life (Shin et al., 2003). In this procedure, the sloshing load on the tank corner in oblique sea is estimated by a geometric average of the maximum impact pressure on the transverse bulkhead and inner-skin bulkhead in from two-dimensional analysis, which is a common practice in the class rules.

Three-dimensional version of the SLOFE2D, SLOFE3D, has been developed jointly by American Bureau of Shipping (ABS) and DSME as a part of ABS-DSME Joint Development Project. SLOFE3D generates sloshing impact load on a specified area of panel for a given tank geometry and motion (Kim et al., 2003). In this paper, experimental and numerical efforts to validate sloshing load calculate by SLOFE3D is presented. Sloshing model test for a three-dimensional membrane-type LNG tank is performed to measure sloshing impact pressure to compare with the numerical value calculated by SLOFE3D. A conventional 140.5K LNG carrier has been selected for the sloshing model test and analysis. ABS selected six regular tank motion for model test conditions, based on seakeeping analysis for tank motion and two-dimensional sloshing analysis following the selection procedure for the critical sloshing wave conditions developed by ABS. For the selected test conditions, sloshing model test has been performed by DSME in Pusan National University, Korea. Geometrically similar LNG tank of 1/70 model scale has been put on a motion platform installed at PNU, which can generate 6-DOF motions. Using the sloshing impact pressure measured from this model test, validation of SLOFE3D has been carried out. The parametric study for SLOFE3D with three different mesh sizes has also been performed.

According to validation study, SLOFE3D shows similar impact pressure magnitude as well as pattern of distribution. However, both SLOFE3D and SLOFE2D seem to underestimate impulse and impact pressure at the height above the upper chamfer.

**MATHEMATICAL FORMULATION**

We consider an irrotational motion of an inviscid and incompressible fluid in a three-dimensional tank in a translation motion. The coordinate system is chosen such that the Z-axis directs against the gravity. The other directions are shown in Fig. 1. The inertial fixed coordinate system is denoted by OXYZ. A moving coordinate system oxzy is also introduced, where the origin is located at the undisturbed free surface. The moving coordinate is translated by the translation vector \( \mathbf{R} = (R_1, R_2, R_3) \) and rotated by roll, pitch and yaw angles, \( \Theta, \Phi \) and \( \Psi \), respectively. The location of the free surface is denoted by \( Z = Z(X, Y, t) \) and \( z = \zeta(x, y, t) \) on the two coordinate systems.
Based on the potential flow theory, the velocity field, \( (U, V, W) \) in the inertial coordinate system \( OXYZ \) can be given by a velocity potential, \( \Phi = \Phi(X, Y, Z, t) \), i.e.,

\[
U = \frac{\partial \Phi}{\partial X}, \quad V = \frac{\partial \Phi}{\partial Y}, \quad W = \frac{\partial \Phi}{\partial Z}
\]  

(1)

The velocity potential satisfies the following governing equation and boundary conditions:

\[
\frac{\partial^2 \Phi}{\partial X^2} + \frac{\partial^2 \Phi}{\partial Y^2} + \frac{\partial^2 \Phi}{\partial Z^2} = 0 \quad \text{in} \quad D,
\]  

(2)

\[
\frac{\partial \Phi}{\partial n} + (\mathbf{R} \cdot \mathbf{r}) \mathbf{n} \quad \text{on} \quad S_w,
\]  

(3)

\[
\frac{\partial Z}{\partial t} + \frac{\partial \Phi}{\partial X} \frac{\partial X}{\partial Z} + \frac{\partial \Phi}{\partial Y} \frac{\partial Y}{\partial Z} - \frac{\partial \Phi}{\partial Z} = 0 \quad \text{on} \quad S_f,
\]  

(4)

\[
\frac{\partial \Phi}{\partial t} + \frac{1}{2} \nabla \Phi \cdot \nabla \Phi + gZ = 0 \quad \text{on} \quad S_p,
\]  

(5)

where \( \mathbf{R} = (R_1, R_2, R_3) \) and \( \mathbf{r} = (x, y, z) \) are the rotational velocity and position vector defined in the moving coordinate system, respectively. It is assumed that the fluid is at rest at the start of tank motion:

\[
\Phi = 0 \quad \text{at} \quad t = 0
\]  

(6)

Since the fluid domain and boundaries keep moving in the inertial coordinate system, it is more convenient to describe the problem in the moving coordinate system. The initial-boundary value problem given by Eqs. (1)-(6) can be reformulated in the moving coordinate as can be seen in Kim et al. (2003).

**FINITE ELEMENT METHOD**

A finite-element method based on Hamilton’s principle has been developed to solve the initial-boundary value problem given by Eqs. (1)-(6). An adaptive mesh system moving vertically to follow the free-surface shape is used to discretize the fluid domain in the LNG tank, as shown in Figure 2. Three-dimensional eight-node iso-parametric brick elements are used. Further details of the numerical scheme can be found in Kim et al. (2003) and Bai & Kim (1995).

**EVALUATION OF PANEL PRESSURE**

Local impact pressure from numerical analysis is sensitive to the numerical parameters such as mesh size and time step. As a result, panel pressure averaged over a prescribed area of tank surface is preferred to local pressure when evaluating sloshing impact pressure. Panel modeling for SLOFE3D has been prepared by DSME in early 2005. The area of each panel used in this validation study is 1 m².

**Paneling of Tank Surface**

The faces of membrane-type LNG tank consist of six rectangular surfaces from longitudinal bulkheads and two vertical surfaces of transverse bulkheads. Faces in longitudinal bulkheads are discretized into square panels of 1 m² area. For transverse bulkheads, parallelograms are used to discretize the upper- and lower-chamfer area as...
shown in Figure 3. Note that some panels are overlapped each other to cover the whole tank surface by panels with the same area.

Paneling of LNG tanks with non-cylindrical shape is more complicated. Figure 4 shows paneling of cylindrical shape (No. 2 tank) and non-cylindrical shape (No. 1 tank).

**Strip Paneling**

Because of large number of panels, detail information such as pressure time history is stored only for limited number of panels. Sometimes it is difficult to determine location of panels where the significant impact pressure is expected. For more systematic selection of critical locations, paneling over specific area where the maximum sloshing impact pressure is expected is preferred to the paneling of the whole tank surface. Two different methods where the paneling is made over transverse and longitudinal strips, respectively, are adopted. The strip paneling is also useful when comparing result of SLOFE3D with SLOFE2D, where the comparison is made for vertical profile of maximum impact pressure. Transverse strips are located at the tank center, the end of forward and aft. Longitudinal strips are located at middle, starboard and portside. Their appearances are shown in Figure 5.

**SLOSHING MODEL TEST**

The 6-DOF-motion platform in Pusan National University has been used for this model test. Figure 6 shows the global appearance of the motion platform. The machinery unit consists of an actuator body, actuator/bracket joints, upper and lower frame, and AC servomotor and driver. The platform is actuated by six actuators and each one is operated by 15 kW electric servomotors. The maximum capacity of the platform is 4 tons and the maximum stroke of each actuator is 1000 mm. The range of displacement and velocity of the motion platform is presented in Table 1.

A 1/70th scale model tank made of 20 mm plexiglas has been used in the model test. The thickness of the model tank is selected to be strong enough to lessen the effect of tank wall vibration. Figure 7 shows the dimensions of model tank. Sloshing impact pressure has been measured at 17 locations. The location of pressure sensors is shown in Figure 8.

Pressure sensors of piezoelectric type produced by Kistler Instrument Corporation are installed on the model tank. The diameter of the measuring surface of the pressure sensor is 9.5mm corresponding to 0.665m in full scale so that the area is approximately a quarter of a single insulation box in full scale. Because the natural frequency of pressure sensor is 70 kHz, any interaction with impact phenomena is not expected. Their sensitivity to acceleration is 0.001bar/g. The measuring range reaches up to 250 bar. However, several bands of measurement range of the pressure sensor can be achieved with the same accuracy. In this model test, the range up to 2.5 bar is utilized and calibrated. The sampling ratio of this model test is 20 kHz that is enough to catch impact phenomena. Figure 9 shows the data acquisition instruments.
Table 1. The range of displacement and velocity

<table>
<thead>
<tr>
<th>Displacement</th>
<th>Velocity</th>
</tr>
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<tbody>
<tr>
<td>Surge -718~716 mm</td>
<td>1700 mm/s</td>
</tr>
<tr>
<td>Sway -655~655 mm</td>
<td>1570 mm/s</td>
</tr>
<tr>
<td>Heave -481~481 mm</td>
<td>1150 mm/s</td>
</tr>
<tr>
<td>Roll -29.1~29.1 deg</td>
<td>70 deg/s</td>
</tr>
<tr>
<td>Pitch -29.4~28.8 deg</td>
<td>69 deg/s</td>
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NUMERICAL RESULTS

Both numerical simulation and sloshing model test have been performed for the 6 motion conditions given in Table 2. The tank motions are selected following ABS sloshing analysis procedure based on three-dimensional seakeeping analysis by NLOAD3D for vessel motion and two-dimensional sloshing analysis by SLOFE2D (ABS, 2006). Numerical results with three different mesh sizes (1.0m, 0.8m, 0.6m) are presented for three-dimensional results, whereas fixed mesh size of 0.4m has been used for two-dimensional calculations. Because of drastic increase of computational time for three-dimensional analysis with finer mesh, three-dimensional simulation with mesh size smaller than 0.6m could not be performed in this study. Convergence study with finer mesh will be presented in the future elsewhere. In figures 10-13, the numerical and experimental values of pressure time history at the selected locations for each filling level are compared. In general, impact pressure calculated by SLOFE3D shows similar magnitude and shape compared with the experiment at the presented filling levels. At filling levels lower than 50%, the numerical value shows less impulse than the measured value. At filling levels higher than 50%, the opposite trend has been observed.

In Figures 14-16, profiles of maximum impact pressure along the portside strip are depicted. Comparisons are made with the numerical results by SLOFE2D and experimental results. For numerical results, the maximum value of panel pressure over 1m² area is presented. Because of
irregular peak values of model test results, average of $1/10^{th}$ highest values are used for the model test results. Also compared are numerical results by SLOFE3D with three different mesh sizes. It can be seen that three-dimensional results are getting closer to the experimental values as the mesh size decreases.

In Figure 17, profiles of maximum impact pressure along the longitudinal strip on tank top are depicted for the filling level of 92.5%. Although the three-dimensional results show convergence at mesh size 0.6m, the results show significant differences with SLOFE2D results and experimental values. The two-dimensional results by SLOFE2D show good agreement along the centerline (Figure 17 (b)), but under predicts the impact pressure at tank corners (Figure 17 (a) & (c)).

CONCLUSIONS
A three-dimensional finite-element code, SLOFE3D, has been developed to simulate sloshing impact pressure in membrane-type LNG tank. To validate the calculated impact pressure, sloshing model test has been performed for regular tank excitation. Comparisons of time history and envelop of impact pressure are made between model test results, two-dimensional results by SLOFE2D and three-dimensional results by SLOFE3D with different mesh sizes. At low filling levels, the numerical results with finer mesh approaches to the experimental value, although convergence of numerical results has not been achieved up to mesh size 0.6m. At high filling level, experimental results show significant three-dimensional effect near tank corner. Numerical results predict the impact pressure at the centerline but underestimate the impact pressure at tank corner. As a concluding remark, more convergence study with finer mesh is required to validate the three-dimensional numerical tool before comparison with experiment is made, which will be pursued in the near future.

<table>
<thead>
<tr>
<th>Filling Level</th>
<th>Motion</th>
<th>Amplitude</th>
<th>Phase</th>
<th>Amplitude</th>
<th>Phase</th>
<th>Amplitude</th>
<th>Phase</th>
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<tbody>
<tr>
<td>40.0%</td>
<td>Lateral</td>
<td>1.54</td>
<td>0.14</td>
<td>0.39</td>
<td>0.12</td>
<td>0.29</td>
<td>0</td>
</tr>
<tr>
<td>40.0%</td>
<td>Vertical</td>
<td>0.51</td>
<td>0.49</td>
<td>0.49</td>
<td>0.49</td>
<td>0.49</td>
<td>0</td>
</tr>
<tr>
<td>35.0%</td>
<td>Lateral</td>
<td>1.56</td>
<td>0.14</td>
<td>0.39</td>
<td>0.12</td>
<td>0.29</td>
<td>0</td>
</tr>
<tr>
<td>35.0%</td>
<td>Vertical</td>
<td>0.51</td>
<td>0.49</td>
<td>0.49</td>
<td>0.49</td>
<td>0.49</td>
<td>0</td>
</tr>
<tr>
<td>30.0%</td>
<td>Lateral</td>
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<td>0.39</td>
<td>0.12</td>
<td>0.29</td>
<td>0</td>
</tr>
<tr>
<td>30.0%</td>
<td>Vertical</td>
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<td>0.49</td>
<td>0.49</td>
<td>0.49</td>
<td>0.49</td>
<td>0</td>
</tr>
<tr>
<td>25.0%</td>
<td>Lateral</td>
<td>1.56</td>
<td>0.14</td>
<td>0.39</td>
<td>0.12</td>
<td>0.29</td>
<td>0</td>
</tr>
<tr>
<td>25.0%</td>
<td>Vertical</td>
<td>0.51</td>
<td>0.49</td>
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<tr>
<td>20.0%</td>
<td>Lateral</td>
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<tr>
<td>20.0%</td>
<td>Vertical</td>
<td>0.51</td>
<td>0.49</td>
<td>0.49</td>
<td>0.49</td>
<td>0.49</td>
<td>0</td>
</tr>
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</table>

Fig. 10 Pressure time history: 16.3% filling ratio, Ch. 15
Fig. 11 Pressure time history: 30% filling ratio, Ch. 15

Fig. 12 Pressure time history: 50% filling ratio, Ch. 11
Fig. 13  Pressure time history: 80% filling ratio, Ch. 7

Fig. 14  Pressure time history: 16.3% Transverse motion, Portside

Fig. 15  Pressure time history: 30% Transverse motion, Portside

Fig. 16  Pressure time history: 50% Transverse motion, Portside
REFERENCES


