COUPLED SEAKEEPING WITH LIQUID SLOSHING IN SHIP TANKS

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ABSTRACT

A three-dimensional forward-speed seakeeping theory is proposed by the inclusion of a tank sloshing flow model. The methodology is based on a coupling model of sloshing and seakeeping in the frequency domain. The fluid forces and moments due to the liquid motion inside tanks are incorporated into the equations of the vessel motion for the coupled seakeeping computations. The classical expression for restoring forces is accordingly modified for the coupled analysis. The global motion responses of a modern LNG carrier at forward speed in beam seas are then calculated to investigate the coupling effects between sloshing and seakeeping. No prominent coupling effect was observed in heave and pitch motions while a significant effect was observed in sway and roll motions. The numerical results show that the proposed theoretical model works well in comparison with experimental results.

INTRODUCTION

Liquid sloshing refers to the resonant fluid motion inside a tank. As the tank moves, it supplies energy to the liquid to sustain the sloshing. Sloshing may induce the violent liquid motion when the period of the tank motion is close to the natural period of the liquid inside the partially filled tank. As the size of the cargo tanks increases, the possibility of the severe sloshing increases, thus pronouncing the severity of potential structural damage. Therefore, the sloshing analysis is of importance to the safety of the tank boundary structures. It is especially of concern for large LNG carriers, large oil tankers, Floating Production, Storage and Offloading (FPSO) systems and Floating Storage & Regasification Units (FSRU). Thus, sloshing is considered to be one of the important factors in the design of these structures.

It is evident that sloshing motions inside tanks interact with vessel motions. In general, sloshing motions have not been included in the seakeeping computations due to their complexity and cumbersomeness. In most seakeeping computations, the liquid inside the tank has been generally treated as a rigid mass. However, theoretical and experimental studies have shown that the interaction effects between sloshing and seakeeping behaviour can be significant to the extent that they can noticeably affect sway and roll motions. The coupling between sloshing and vessel motion can change the magnitude of sloshing pressure on the tank structures and, in return, the vessel motion can also be changed.

The coupling effects between sloshing and seakeeping have been studied by numerical calculations and experimental measurements. To name a few investigators, Francescutto and Contento (1994) presented the experimental investigation results of the coupling between roll motion and sloshing in a regular beam sea condition. Journée (1997) studied the effect of liquid cargo on ship motions based on the calculations and experiments. Molin et al. (2002) proposed a theoretical model for the combined dynamics of a floating body and sloshing motions in internal tanks. They presented the comparison of results between calculations and measurements of barge motion and free surface elevation in the tanks. Rognebakke and Faltinsen (2003) investigated experimentally and theoretically the coupling effect for a rectangular hull section excited in sway by regular waves. They found that the damping of the sloshing motion in a certain frequency range plays an important role in determining the calculated coupled motion. Malenica et al. (2003) proposed a thorough theoretical model of dynamic seakeeping/sloshing coupling, which is formulated in the frequency domain. Gaillarde et al. (2004) presented the model tests and numerical results obtained for an LNG FPSO and an LNG carrier with partially filled tanks. Their experimental measurements show significant coupling effect on the roll motion due to sloshing. Newman (2005) presented the numerical results of the WAMIT panel code...
for the linearly coupled sloshing and vessel motions in waves. In certain circumstances, the time-domain sloshing simulation codes have been incorporated into the time-domain ship motion solvers in order to investigate the sloshing impact load on tank walls. This type of time-domain approach is more desirable for accurate evaluation of coupling effects at severe sea states. These efforts can be found, for example, in Kim et al. (2003, 2007), Kim et al. (2005), Cho et al. (2007) and Lee et al. (2007, 2008).

In this paper, a three-dimensional forward-speed seakeeping theory is enhanced by considering the liquid motion effect on vessel motions. The dynamic coupling model of sloshing and seakeeping is formulated in the frequency domain. Special attention is paid to the linearization of the free surface condition for the sloshing flow by introducing damping effect. To identify natural frequencies of the sloshing tank at the given filling depth, an eigenvalue problem is established first. The natural frequencies of the liquid motions in the tank are then determined on the basis of the linear theory of water waves by three-dimensional boundary element solutions. Next, a boundary value problem is established to determine the added mass and damping forces due to the liquid sloshing motion. For numerical computations, the linearized free-surface boundary condition introducing damping terms is used while no flux condition is applied for the tank walls. To validate the theoretical model developed here, the global motion responses of a modern LNG carrier at forward speed in beam seas are calculated and compared with experimental results. The importance of coupling effects between sloshing and seakeeping is finally addressed.

SLOSHING

Consider the fluid motion inside the partially filled oscillating tank. The Cartesian coordinate system O-xyz fixed to the tank is used. Its origin O coincides with the middle point of the undisturbed free surface inside the tank. The z-axis is vertical and points upward against gravity, and the undisturbed free surface is taken as the plane z=0. We assume the tank motions are small and harmonic in time so that the resulting fluid motion inside the tank can also be small. The unsteady potential \( \phi \) satisfying the Laplace equation can then be determined from the following boundary conditions (Faltinsen et al., 2003)

\[
\frac{\partial \phi}{\partial n} = \tilde{v}_0 \cdot \tilde{n}_T + \tilde{\Omega}_T \cdot [\tilde{r}_T \times \tilde{n}_T] \quad \text{on } S_T \tag{1}
\]

and

\[
\frac{\partial \phi}{\partial n} = \tilde{v}_0 \cdot \tilde{n}_T + \tilde{\Omega}_T \cdot [\tilde{r}_T \times \tilde{n}_T] + \frac{\partial \eta}{\partial t} \quad \text{on } S_r \tag{2}
\]

\[
\frac{\partial \phi}{\partial t} = g \theta x + g \phi y + g \eta = 0 \quad \text{on } S_r \tag{3}
\]

where the time-dependent vectors \( \tilde{v}_0 = (v_{0x}, v_{0y}, v_{0z}) \) and \( \tilde{\Omega}_T = (\tilde{\phi}, \tilde{\theta}, \tilde{\psi}) \) represents the translational and angular velocities of the O-xyz coordinates relative to an absolute coordinate system, respectively, \( \tilde{r}_T \) is the position vector of the point relative to the origin O, \( g \) is the gravitational acceleration, the unit normal vector \( \tilde{n}_T \) points out of the fluid region, \( \eta \) is the free surface elevation and \( t \) represents the time. Recalling the free surface is horizontal and substituting Eq. (3) into Eq. (2), the combined free surface condition becomes

\[
\frac{\partial \phi}{\partial z} + \frac{1}{g} \frac{\partial^2 \phi}{\partial t^2} + \mu_1 \frac{\partial \phi}{\partial t} + \mu_2 \phi = v_{0z} \quad \text{on } S_r \tag{4}
\]

where the parameters \( \mu_1 \) and \( \mu_2 \) are introduced to consider the hydrodynamic damping due to the viscosity of the fluid. With assumption of the linearized theory, the unsteady potential \( \phi \) can be linearly decomposed as follows:

\[
\phi = \sum_{j=1}^{6} \xi_j^T \phi_j e^{i\omega t} \tag{5}
\]

where \( i = \sqrt{-1} \) and \( \phi_j \) is the potential related to the tank motions \( \xi_j^T \) in the j-mode, \( \omega \) is the circular frequency of the tank motion. Substituting Eq. (5) into the boundary conditions (1) and (4), we obtain

\[
\frac{\partial \phi_j}{\partial n} = i \alpha \xi_j \quad \text{on } S_T \tag{6}
\]

and

\[
\frac{\partial \phi_j}{\partial z} + (v - i \alpha \mu_1 - \mu_2) \phi_j = i \alpha \xi_j \quad \text{on } S_r \tag{7}
\]

where \( v = \omega^2 / g \) and the normal vector \( n_j \) is defined as \((n_1, n_2, n_3) = \tilde{n}_T \) and \((n_x, n_y, n_z) = \tilde{r}_T \times \tilde{n}_T \). The forcing term \( f_j \) can be given by \( f_{1x} = 0, f_{1y} = 0, f_{1z} = 1, f_2 = 0, f_3 = 0 \) and \( f_4 = 0 \).

Applying Green’s theorem to the fluid region inside the tank, the velocity potential \( \phi_j \) can be transformed into the integral form as follows:

\[
\phi_j (p) = \iint_{S_T} \left( \sum_{i=1}^{6} \frac{\partial G}{\partial n_i} - G \frac{\partial \phi_i}{\partial n} \right) dS \tag{8}
\]

Here, we use the form of Green function G given by Rankine source potential. Imposing the boundary conditions (6) and (7), the boundary integral equation becomes

\[
2 \pi \phi_j (p) - \iint_{S_T} \phi_i \left( \frac{1}{R} \right) dS - \iint_{S_r} \left( \frac{1}{R} \right) \nabla \cdot \phi_i dS = -i \alpha \left[ \iint_{S_T} n_j \left( \frac{1}{R} \right) dS + \iint_{S_r} f_i \left( \frac{1}{R} \right) dS \right] \tag{9}
\]

where \( \nabla = v - i \alpha \mu_1 - \mu_2 \) and R is the distance between the field point \( p(x,y,z) \) due to a source at the point \( q(\xi, \eta, \zeta) \), which is given by

\[
R = \sqrt{(x - \xi)^2 + (y - \eta)^2 + (z - \zeta)^2} \]

The potential \( \phi \) on the
mean wetted part of the tank surface $S_T$ and undisturbed free surface $S_P$ is the unknown to be determined in Eq. (9). To obtain the numerical solution, the integral equation (9) is approximated in a discretized form by dividing wetted part of the tank surface and the undisturbed free surface inside the tank into a number of panels. Assuming the singularity strengths are constant on each panel, Eq. (9) leads to a linear system of the complex simultaneous equations for the unknown strengths. Once the singularity strengths $\Phi_i$ are determined, fluid velocities and pressure are to be computed.

The fluid forces and moments acting on the tank can be obtained by the integration of the pressure $p$ over its instantaneous wetted tank surface $\tilde{S}_T$. Thus, we have

$$\tilde{F}^T = - \int_{\tilde{S}_T} (p - p_a) \tilde{n}_i dS \quad (10)$$

$$\tilde{M}^T = - \int_{\tilde{S}_T} (p - p_a)(\tilde{r}_i \times \tilde{n}_i) dS \quad (11)$$

where $p_a$ is the atmospheric pressure, $\tilde{r}_i$ is the instantaneous position vector of the point on $\tilde{S}_T$ relative to the center of tank rotation, which is the origin O and $\tilde{n}_i$ denotes the instantaneous unit normal vector directed outside the fluid region. Applying Bernoulli’s equation, the pressure anywhere in the fluid region can be obtained. Since the forces and moments in a linear sense are of interest, the pressure on the instantaneous wetted part of the tank surface is expanded into a Taylor series about the mean wetted surface. Retaining up to the linear terms for $\Phi$, the pressure on the instantaneous wetted surface can be expressed as

$$p - p_a = -\rho_T (\Phi + gz) + O(\Phi^2) \quad (12)$$

where $\rho_T$ is the density of the fluid inside the tank. This expression can be further expanded into a Taylor series about the mean wetted part of the tank surface $S_T$ as

$$p|_{\tilde{S}_T} = p|_{S_T} + \alpha_T \cdot \nabla p|_{S_T} + O(\alpha_T^2) \quad (13)$$

where $\alpha_T$ is the local oscillatory displacement vector of the tank’s surface for the small motions, which is given by

$$\alpha_T = \tilde{\alpha}_T + \tilde{\Omega}_T \times \tilde{r}_i \quad (14)$$

and it leads to

$$p - p_a = -\rho_T (\Phi + gz) - \rho(\tilde{\alpha}_T \cdot \nabla)(gz) \quad \text{on} \quad S_T \quad (15)$$

The integral over the instantaneous wetted surface $\tilde{S}_T$ is also expanded about the mean wetted surface of the tank surface $S_T$. The normal vectors can also be expressed in a Taylor expansion as

$$\tilde{n}_i = \tilde{n}_i + \tilde{\Omega}_T \times \tilde{n}_i \quad (16)$$

$$\tilde{r}_i = \tilde{r}_i + \tilde{\Omega}_T \times \tilde{r}_i \quad (17)$$

$$\tilde{\alpha}_T \times \tilde{n}_i = (\tilde{\alpha}_T \times \tilde{r}_i) \times \tilde{n}_i + \tilde{r}_i \times (\tilde{\Omega}_T \times \tilde{n}_i) \quad (18)$$

Substituting these equations into Eqs. (10) and (11), and rearranging up to the first order, we obtain

$$\tilde{F}^T = \rho_T \int_{S_T} \Phi \tilde{n}_i dS + \rho_T g \int_{S_T} \tilde{z}\tilde{n}_i dS$$

$$+ \rho_T g \int_{S_T} z(\tilde{\alpha}_T \cdot \nabla)\tilde{n}_i dS \quad (19)$$

$$\tilde{M}^T = \rho_T \int_{S_T} (\Phi \tilde{r}_i \times \tilde{n}_i) dS + \rho_T g \int_{S_T} z(\tilde{\alpha}_T \times \tilde{n}_i) dS$$

$$+ \rho_T g \int_{S_T} z(\tilde{\Omega}_T \times \tilde{r}_i) \times \tilde{n}_i + \tilde{r}_i \times (\tilde{\Omega}_T \times \tilde{n}_i) dS \quad (20)$$

The first terms in Eqs. (19) and (20) are the hydrodynamic components due to liquid dynamic action. These parts can be separated into the added mass and damping terms. The added mass and damping forces in the $i$-th mode due to oscillation in the $j$-th mode are then

$$-\omega^2 A_{ji}^T + i \omega B_{ji}^T = \rho_T \int_{S_T} (i \omega \Phi) \tilde{n}_i dS \quad (21)$$

The remaining parts in Eqs. (19) and (20) are the hydrostatic forces and moments. Since the heave motion of the tank does not change the hydrostatic pressure, the instantaneous vertical coordinate $z$ can be expressed as

$$z = z' + \xi_T x' + \xi_T y' \quad (22)$$

where $(x', y', z')$ denotes the body fixed coordinates of $\tilde{z}$ for the same $\tilde{x} = (x, y, z)$ at rest. Integrating the static part from Eqs. (19) and (20) yields the following expressions

$$\tilde{F}_{\text{hn}}^T = -\rho_T g [\nabla_T \cdot (\xi_T S_T^2 + \xi_T S_T^2)] \tilde{k} \quad (23)$$

$$\tilde{M}_{\text{hn}}^T = -\rho_T g [\frac{1}{2} \nabla_T \cdot (S_T^2)] \tilde{k} \quad (24)$$

where $\nabla_T$ is the volume of the fluid inside the tank, and $(x'_h, y'_h, z'_h)$ is defined as

$$x'_h, y'_h, z'_h = \frac{1}{2} \int_{S_T} \tilde{x}_i \tilde{n}_i dS \quad \text{for} \ i = 1, 2, 3 \quad (25)$$

with the notation $(x_1, x_2, x_3) = (x, y, z)$. The moments over the water plane inside the tank are defined as follows:

$$S_T = \int_{S_T} x_i dS \quad S_T = \int_{S_T} \tilde{x}_i dS \quad \text{for} \ i = 1, 2 \quad (26)$$

Here, it should be noted that $S_T^1 = S_T^2 = 0$ due to the fact that the origin of the tank rotation is the center of the free surface.

In general, the forces and moments due to the weight of the liquid inside the tank are considered in the ship restoring matrix. Therefore, this contribution should be subtracted from Eqs. (23) and (24), which is given by

$$\tilde{F}_{\text{hn}}^T = -m_T g \tilde{k} \quad (27)$$

$$\tilde{M}_{\text{hn}}^T = -m_T g [\frac{1}{2} \nabla_T \cdot (x'_h, y'_h, z'_h)] \quad (28)$$

where $m_T$ is the mass of liquid in the tank and $(x'_h, y'_h, z'_h) = (x, y, z)$ is the center of gravity of the liquid.
the resulting total forces and moments from the liquid in the tank can be written as
\[ F^T_i = -\rho_i x^T_{y^T} - m_i y^T_{z^T} \delta_{i3} - \rho_i g x^T_{z^T} - m_i y^T_{y^T} \delta_{i4} + (\rho_i g x^T_{y^T} x^T_{z^T} - m_i g x^T_{y^T} y^T_{z^T}) \delta_{i1} \]
\[ + (\rho_i g y^T_{y^T} x^T_{z^T} - m_i g y^T_{y^T} y^T_{y^T}) \delta_{i2} + i\omega_i T^T_{y^T} - i\omega_i B^T_{y^T} - C^T_{ij} \xi_{ij} \]  
where the Kroenecker delta function \( \delta_{ij} = 1 \) when \( i = j \) and 0 otherwise. Recalling that the positions of the center of gravity and the center of buoyancy for the liquid of the tank are the same, and the mass of the liquid is equal to the displaced volume, the resulting forces and moments from the liquid in the tank can be written as
\[ F^T_i = [i\omega_i T^T_{y^T} - i\omega_i B^T_{y^T} - C^T_{ij} \xi_{ij}] \]
where the expression for the restoring coefficient \( C_{ij} \) can be given by
\[ C_{ij} = -\rho_i g \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & S^T_{11} & S^T_{12} & 0 \\ 0 & 0 & S^T_{21} & S^T_{22} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \]  
(31)
It is noted that the coefficients of restoring matrix are negative. This proves the well-known fact that the free surface effect due to the liquid motion in the partially filled tank reduces the stability of the vessel. The derivation process used here is similar to the one used in deriving the classical linear hydrostatic forces and moments for a freely floating body (Newman, 1977). So far, the forces and moments are described in the tank local coordinate system, therefore, we need to define the forces and moments into the ship global coordinate system. The following relations can be defined between the two coordinate systems (Malenica et al., 2003)
\[ \xi^T = \xi + \Omega \times \vec{r}_{TG}, \quad \Omega = \Omega \]
\[ \vec{F}^T = \vec{F}, \quad M^T = M + \vec{F} \times \vec{r}_{TG} \]  
(32)
(33)
where \( \xi^T \) and \( \Omega \) are the ship translational and rotational motions of the ship defined in the global coordinates, \( \vec{F} \) and \( M \) are the force and moment components in the global coordinate system, and \( \vec{r}_{TG} = (x_{TG}, y_{TG}, z_{TG}) \) is the position vector of the origin of the tank local coordinates (tank rotation) relative to the center of ship motion. Using the relations above, Eq. (30) can be written as
\[ F^T_i = [i\omega_i (A^T_{ij} + A^T_{y^T}) - i\omega_i (B^T_{ij} + B^T_{y^T}) - (C^T_{ij} + C^T_{y^T})] \xi_{ij} \]  
(34)
where
\[ A^T_{ij} = \begin{bmatrix} 0 & -\alpha_{ij} T^T_{y^T} \\ -\alpha_{ij} T^T_{y^T} & 0 -\alpha_{ij} T^T_{y^T} \\ 0 & -\alpha_{ij} T^T_{y^T} & 0 -\alpha_{ij} T^T_{y^T} \end{bmatrix} \]  
(35)
(36)
with the notation \( (x_{TG}, y_{TG}, z_{TG}) = (x_T - x_G, y_T - y_G, z_T - z_G) \) and \( \alpha_{ij} \) is the 3x3 submatrix of the added mass matrix \( A^T_{ij} \), which can be expressed as
\[ A^T_{ij} = \begin{bmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{bmatrix} \]  
(37)
The damping matrix \( B^T_{ij} \) and resorting matrix \( C^T_{ij} \) due to the coordinate transformation can also be obtained in similar way with the added mass matrix \( A^T_{ij} \).

**SEAKEEPING**

Consider a ship advancing at the forward speed \( U \) in a regular wave. The equations of motion for oscillations of the ship can be derived by equating the sum of the hydrodynamic forces acting on the hull and tank surfaces to the inertial forces associated with the accelerations of the ship. The six components of the inertial force can be written as
\[ \sum_{j=1}^{6} M_{ij} \ddot{x}_{ij} = F_i \quad \text{for} \quad i = 1, 2, \ldots, 6 \]  
(38)
where \( \ddot{x}_{ij} \) is the six rigid-body global motions and the matrix \( M_{ij} \) denotes the mass-inertia coefficient. Assuming the linearized small motions, the fluid force \( F_i \) can be linearly decomposed into four components as
\[ F_i = F_i^V + F_i^R + F_i^D + F_i^T \]  
(39)
where \( F_i^V, F_i^R, F_i^D \) and \( F_i^T \) denotes restoring, radiation, wave exciting and sloshing liquid forces, respectively. The fluid force and moment acting on the ship hull can be obtained by the integration of the pressure over the yet unknown instantaneous hull wetted surface. Since the force and moment in a linear sense are of interest, the pressure on the instantaneous wetted part of the hull surface is expanded into a Taylor series about the mean wetted position. The pressure on the mean wetted part of the hull surface can be expressed as
\[ p - p_s = -\rho (\dot{\Phi} + \frac{1}{2} \nabla \Phi \cdot \nabla \Phi \cdot \frac{1}{2} U^2 + \nabla \Phi \cdot \nabla \Phi + gx) \]  
(40)
on \( S \)
where \( p \) is the density of the surrounding fluid, \( \Phi(\vec{x}) \) is the velocity potential due to steady flow part, \( \phi(\vec{x},t) \) is due to the unsteady flow part and \( \vec{a} \) is the local oscillatory displacement vector of the ship for the small motions. The integral over the instantaneous wetted surface is also expanded about the mean wetted surface of the hull and differential surface bounded by the wave elevation and the oscillatory displacement.
We assume that the ship is in a stable equilibrium position of the ship advancing with constant speed in waves, the steady force and moment acting on the ship hull and tank surfaces are balanced by the weight of the ship and its moment about the center of rotation. The forces and moments obtained from the integration of pressure are then separated into static and dynamic components. The static force consists of the hydrostatic and steady flow
parts. The steady flow that is associated with the term $\nabla \Phi$ in Eq. (40) also contributes to the static force. After adding the corresponding forces and moments due to the weight of the ship including the mass of the liquid in tanks, the resulting total static force can be expressed as given in Kim and Shin (2007):

$$ F_0^r = (\rho \nu - m)g\delta_3 + (\rho g x_n - m g x_n) \delta_s - (\rho g x_n - m g x_n) \delta_s $$

where $\forall$ is the displaced volume at the mean wetted position, $m$ is the total ship mass, $(x_n, x_n, z_n)$ is the position vector of the point on $S$ is the position, $m$ is the total ship mass, $(x_n, x_n, z_n)$ is the center of buoyancy and $(x_n, x_n, z_n)$ is the center of gravity of the ship. The restoring coefficient $C_{ij}$ is then given as

$$ C_{ij} = \rho g \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & A_{ij} & S_1 & -S_1 & 0 \\ 0 & 0 & S_2 & S_1 & -S_1 & 0 \\ 0 & -S_1 & S_2 & S_1 & -S_1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} $$

for $i=1,2,3$ and

$$ C_{ij} = \rho g \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & A_{ij} & S_1 & -S_1 & 0 \\ 0 & 0 & 0 & S_2 & S_1 & -S_1 \\ 0 & 0 & -S_1 & S_2 & S_1 & -S_1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} $$

for $i=4,5,6$. Here, $\vec{e}_i$ and $\vec{y}_j$ are the unit vectors in $x$, $y$ and $z$ direction, respectively, and $\vec{b}_j$ is defined as

$$ \vec{b}_j = \vec{e}_{j-3} \times \vec{y}_j $$

The unit normal vector $\vec{n}$ on $S$ is directed inside the fluid region and $\vec{r}$ is the position vector of the point on the mean wetted surface relative to the center of the vessel rotation. $A_{ij}$ is the water plane area of the ship and the moments over its water plane surface are given by

$$ S_i = \int x_i \, dS, \quad S_j = \int x_i \, x_i \, dS \quad \text{for} \ i=1,2 $$

It is noted that the resulting restoring forces and moments for the forward-speed seakeeping problem require the solution of the steady problem. When the basis flow is approximated by the free stream, i.e., $\Phi = -\vec{U}_x$, the restoring force coefficient (42) reduces to that of the classical freely floating body case. Namely, it becomes

$$ F_0^r = \rho g \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & A_{ij} & -S_1 & 0 & 0 \\ 0 & 0 & S_2 & S_1 & 0 & 0 \\ 0 & -S_1 & S_2 & S_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} $$

This restoring matrix can also be used when the trim and sinkage achieved at the forward-speed steady equilibrium position is small.

The remaining part is the hydrodynamic component which can be separated into the radiation and wave exciting forces. The radiation forces can be obtained from the forced motion potential. The radiation force in the $i$-th mode due to oscillation in the $j$-th mode are given as

$$ F_i^o = \omega^2 A_{ij} - io B_{ij} = \rho \int_{S} \nabla \phi \cdot n \, dS $$

where $A_{ij}$ and $B_{ij}$ are the added mass and damping coefficients, respectively. The wave exciting forces are due to the incident waves and diffracted waves around the restrained ship. Thus, the wave exciting forces in the $i$-th mode are given as

$$ F_i^o = \rho \int_{S} \left( \nabla \phi \cdot \nabla \phi + \nabla \phi \cdot \nabla \phi \right) \cdot n \, dS $$

Rearranging the above equations, we obtain the equations of motion for the coupled analysis of seakeeping and sloshing

$$ \sum_{j=1}^{6} \left( -\omega^2 (M_{ij} + A_{ij} + A_{ij}^T) + io (B_{ij} + B_{ij}^T + B_{ij}^T) \right) $$

where the mass-inertia matrix coefficient of the light ship $M_{ij}$ is given by

$$ M_{ij} = \begin{bmatrix} \vec{m} & 0 & 0 & 0 \vec{m} \vec{x}_G & 0 \\ 0 & \vec{m} & 0 & -\vec{m} \vec{x}_G \vec{x}_G & 0 \vec{m} \vec{x}_G \\ 0 & 0 & \vec{m} & 0 & -\vec{m} \vec{x}_G \vec{x}_G & 0 \vec{m} \vec{x}_G \\ 0 & 0 & -\vec{m} \vec{x}_G \vec{x}_G & 0 & \vec{m} & 0 \vec{m} \vec{x}_G \\ 0 & 0 & -\vec{m} \vec{x}_G \vec{x}_G & 0 & \vec{m} & 0 \vec{m} \vec{x}_G \\ 0 & 0 & -\vec{m} \vec{x}_G \vec{x}_G & 0 & \vec{m} & 0 \vec{m} \vec{x}_G \end{bmatrix} $$

Here, $\vec{m}$ is the mass of the light ship and $\vec{I}_k$ is the inertia matrix coefficient of the light ship with respect to the center of vessel rotation $(x_G, y_G, z_G)$ including the mass of fluid in the tanks. To determine the hydrodynamic coefficients $A_{ij}$, $B_{ij}$ and $F_{ij}$, the approximate solution method which uses the zero-speed pulsating Green function for deep water is adopted as described in Kim (2005) with the choice of the uniform flow as a steady flow model.
**TANK RESONANCE PERIOD**

The resonant condition of the tank depends on the tank shape, tank size, filling level and configuration of internal structural members as well as the mode of the tank motion. When the motion of the tank is sinusoidal and the fluid motion becomes very violent at the specific period, we call the relevant period of the fluid motion the resonance period. In general, the determination of the resonance period requires an experiment or numerical simulation. Although there are an infinite number of the resonance periods, the fundamental period is primarily significant for applications. To calculate the natural periods of the tank at the specific filling level, an eigenvalue problem for the prediction of the sloshing resonance period can be established and solved numerically. Here, we establish the boundary value problem in three-dimensional fluid domain. The governing equation is the Laplace equation, which is

$$ \nabla^2 \phi = 0 \quad \text{in the fluid region} \quad (53) $$

The boundary condition for the tank walls is the no-flux condition, which is

$$ \frac{\partial \phi}{\partial n} = 0 \quad \text{on the mean wetted part of tank walls} \quad (54) $$

On the free surface, the linearized free-surface boundary condition is applied, that is

$$ \frac{\partial \phi}{\partial z} - \frac{\omega^2}{g} \phi = 0 \quad \text{on the undisturbed free surface} \quad (55) $$

The potential on the free surface can be expanded as

$$ \phi(s) = \sum_i \phi_i \Phi_i(s) \quad (56) $$

where $\Phi_i(s) = 1$ at $s = s_i$, otherwise $\Phi_i(s) = 0$ and $s$ denotes the free-surface region. Taking the derivative of Eq. (56) in the z-direction, it becomes

$$ \frac{\partial \phi}{\partial z} = \sum_i \phi_i \frac{\partial \Phi_i(s)}{\partial z} = \sum_i K_{ij} \phi_j \quad (57) $$

where $K_{ij}$ is defined as

$$ K_{ij} = \frac{\partial \Phi_i(s)}{\partial z} \quad (58) $$

Substituting Eq. (57) into Eq. (55), it results in

$$ \left[ \frac{\omega^2}{g} I - K_{ij} \right] \phi = 0 \quad (59) $$

where $I$ is the identity matrix. It can be easily shown that the eigenvalues of $K_{ij}$ are related to the resonance frequencies. Thus, we have

$$ \lambda_i = \frac{\omega_n^2}{g} \quad (60) $$

where $\lambda_i$ are the eigenvalues of $K_{ij}$ and $\omega_n$ are the natural frequencies of i-th corresponding mode.

Based on a direct approach (source and dipole distributions formulation), the boundary value problem described above can be transformed into a boundary integral form. The integral equation is then approximated in a discretized form by dividing the wetted part of the tank walls and free surface into quadrilateral panels. It is assumed that the singularity strengths are constant on each panel. The collocation points where the discretized integral equation is satisfied are the centroids of each panel and the influence coefficients of the Rankine terms over the panel are evaluated analytically using the method of Hess and Smith. The lower and upper triangular matrices (LU) decomposition solver is used to determine the unknown singularity strength (potential). To investigate and verify the accuracy of the developed code here, the resonance periods of the three-dimensional rectangular tank with square base are calculated and compared with the analytical solutions. For numerical computations, the wetted part of the tank walls and free surface are discretized into a number of panels, as shown in Fig. 1.

![Fig. 1 Example of panel arrangement for sloshing resonance period prediction of rectangular tank with square base](image)

As can be seen in Fig. 2, the calculated natural periods depending on the filling depth is in excellent agreement with the analytical solutions. In principle, the sloshing natural periods of any arbitrarily-shaped three-dimensional internal tanks can be calculated.

![Fig. 2 Sloshing resonance periods of rectangular tank with square base](image)
COUPLING EFFECT

To investigate the sloshing effect on vessel motion responses, a modern LNG carrier is selected. Seakeeping tests were carried out in the MARIN Seakeeping and Maneouvrering Basin as part of the ‘Seakeeping of structures Affected by Liquid Transient’ Joint Industry Project (SALT JIP) (MARIN, 2003). The model was built on a geometric scale ratio of 1:50. The model was equipped with two prismatic tanks filled with fresh water and bilge keels. The main particulars of the LNG carrier and its cargo tanks are summarized in Table 1.

Table 1 Main particulars of LNG carrier and its tanks

<table>
<thead>
<tr>
<th>LNG Carrier</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Length Lpp</td>
<td>m 274.0</td>
</tr>
<tr>
<td>Breadth moulded B</td>
<td>m 44.20</td>
</tr>
<tr>
<td>Draft T</td>
<td>m 11.58</td>
</tr>
<tr>
<td>Displacement volume</td>
<td>m³ 103,019.2</td>
</tr>
<tr>
<td>Block Coefficient Cn</td>
<td>- 0.735</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Forward Tank</th>
<th></th>
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</thead>
<tbody>
<tr>
<td>Length L_T</td>
<td>m 47.18</td>
</tr>
<tr>
<td>Breadth B_T</td>
<td>m 39.10</td>
</tr>
<tr>
<td>Height H_T</td>
<td>m 26.86</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Aft Tank</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Length L_T</td>
<td>m 41.40</td>
</tr>
<tr>
<td>Breadth B_T</td>
<td>m 39.10</td>
</tr>
<tr>
<td>Height H_T</td>
<td>m 26.86</td>
</tr>
</tbody>
</table>

Three different filling levels were considered during the model tests. Their experimental conditions are summarized in Table 2, together with the calculated tank resonance periods and the number of panels used for the sloshing natural period calculations.

Table 2 Numerical parameters used in the computations

<table>
<thead>
<tr>
<th>Tank</th>
<th>Filling Level (% of H_T)</th>
<th>Resonance Period (sec)</th>
<th>Number of Panel</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>T_n1</td>
<td>T_n2</td>
</tr>
<tr>
<td>AFT</td>
<td>R0</td>
<td>0.000</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>R1</td>
<td>18.615</td>
<td>12.63</td>
</tr>
<tr>
<td></td>
<td>R2</td>
<td>37.230</td>
<td>9.21</td>
</tr>
<tr>
<td></td>
<td>R3</td>
<td>55.845</td>
<td>8.09</td>
</tr>
<tr>
<td>FWD</td>
<td>R0</td>
<td>0.000</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>R1</td>
<td>18.615</td>
<td>14.31</td>
</tr>
<tr>
<td></td>
<td>R2</td>
<td>37.230</td>
<td>10.33</td>
</tr>
<tr>
<td></td>
<td>R3</td>
<td>55.845</td>
<td>8.95</td>
</tr>
</tbody>
</table>

The filling levels R1, R2 and R3 denote 18.615%, 37.230% and 55.845% of the corresponding tank height, respectively. The draft and vertical position of center of gravity of the ship are kept constant at each filling condition, which is consistent with the model tests. The mean wetted part of the tank walls and its free surface at different filling conditions are discretized into a number of panels for numerical computations, as shown in Fig. 3.

Fig. 3 Panel arrangement of forward tank at different filling level conditions

Figures 4 through 9 show the calculated added mass and damping of the forward tank at the filling level R2 depending on the value of the damping parameter used in the free surface condition (4). The damping parameters $\mu_1$ and $\mu_2$ are taken as $\mu_1 = 2 \omega \kappa$ and $\mu_2 = \mu_1 ^2 / 4$. At the resonance periods, it is clearly seen that the added mass can reach $\pm \infty$. As expected, the effect of the damping value is prominent in diminishing the added mass and damping near the sloshing resonance periods. In general, the damping parameters have influence on limiting the peak values of the added mass near the resonance periods. To find the proper value of the damping parameter, the experimental measurements can be used to estimate it by considering the energy dissipation inside the sloshing tank. It is noted that the heave damping force is relatively small compared to those of sway and roll, as can be seen in Fig. 8.

Fig. 4 Sway added mass of forward tank
Figures 10 through 15 show the added mass and damping of the forward tank for the three different filling levels. The damping parameter \( \kappa \) used here is 0.002. The sloshing natural frequencies can be clearly identified where the rapid change of the added mass occurs. As summarized in Table 2, these frequencies (periods) match the calculated values based on three-dimensional boundary element solutions of the eigenvalue problem. Another observation is that the sloshing natural frequency increases as the filling level increases as expected. As pointed out in Newman (2005), the heave added mass is equal to \((1-g/\omega^2d)\) times the fluid mass for a tank with the filling depth \(d\). The numerical results shown in Fig. 11 confirm this observation. Figure 15 shows that the damping in roll becomes small for the higher filling level while it is not true for sway mode as shown in Fig. 13. Regardless of the filling level, the damping can be negligible for heave mode, as shown in Fig. 14.
Coupled Seakeeping with Liquid Sloshing in Ship Tanks

Figure 10 shows the sway added mass of the forward tank at different filling levels. Figure 11 illustrates the heave added mass of the forward tank at different filling levels. Figure 12 represents the roll added moment of the forward tank at different filling levels. Figure 13 depicts the sway damping force of the forward tank at different filling levels. Figure 14 shows the heave damping force of the forward tank at different filling levels. Figure 15 illustrates the roll damping moment of the forward tank at different filling levels.

Figure 16 shows one of the examples of the panel arrangement for the seakeeping calculations. The beam sea condition is selected for the numerical calculation, where the coupling effects are known to be most evident.
The ship speed used is 20 knots. The mean wetted part of the hull surface is discretized with 802 panels and the interior free surface inside the ship hull with 172 panels to suppress the so-called irregular frequency phenomenon. The number of panels on the tank wetted surface and its free surface used for the numerical calculations are given in Table 2. Figure 17 shows the results of sway motion responses. Due to the coupling effect, the peak of the sway motion appears near the natural frequency of sway mode and the sway motion decreased at the frequencies lower than the resonance frequency. At the frequencies higher than the combined resonance frequency, the sway motion increases due to the coupling effect. Overall, the predicted sway motion is underestimated at the peak compared to the experimental observation. As can be seen in Fig. 18, the heave response is not affected by the liquid sloshing action in the tanks and no prominent coupling effect was observed. Similar observation was also found in the results of the pitch motion response. As shown in Fig. 19, it is evident that the double peaks appear in roll motion response. The coupling effect is more significant at the higher filling level condition where the peak values of the roll motion decreases. It is found that the coupling effect was most prominent in roll motion.

In summary, no prominent coupling effect was observed in heave and pitch motions but a significant effect was observed in sway and roll motions. As shown in Fig. 19, the higher filling level results in decreasing the roll motions. Overall, a good agreement between the predicted and the measured motions was obtained.

CONCLUSIONS

The sloshing effect on the ship global motion responses has been investigated using a three-dimensional panel method formulated in the frequency domain. The global motion responses of a LNG carrier at forward speed in beam seas have been compared with the available experimental measurements. The numerical results have illustrated the importance of the coupling effects between seakeeping and liquid motions inside tanks. The numerical results show that the proposed theoretical model works well in comparison with the experimental results.
ACKNOWLEDGMENTS

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REFERENCES