Taylor Series applied for Probabilistic Presentation of the Geometric Properties of Structural Profiles

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ABSTRACT

The paper discusses the results of a project aimed to probabilistic presentation of the geometric properties of shipbuilding structural profiles when assessing elastic bending strength. The derived equations are used to calculate the probability of exceeding permissible limits of certain geometric properties.

Taylor series approximation method is used in the work as a basic method. Also, another simpler approximate method is suggested with sufficient accuracy. The proposed methods can be used in the initial design stage of the ship structure. They allow for comparison between different structural profiles using as a criterion the probability that these structural profiles will continue to meet given renewal criteria requirements for stiffeners and girders throughout the ship’s lifetime. The methods can also be used for better planning of ship structure’s maintenance and repair using as a criteria the probability that in some given time the stiffeners/girders will fail to meet the requirements of given Renewal Criteria.

I. INTRODUCTION

The application of the Time-dependent Reliability Theory into ship structures’ design, maintenance and repair requires presentation of the external and internal loads, geometric properties of the hull girder and its components, mechanical properties of the material as random functions of time. One of the least explored parameters in this process is the presentation of the geometric properties of the structure as random functions of time. Therefore the author addressed this issue and developed methods that could be of help when Time-dependent Reliability Theory is applied in shipbuilding.

The probabilistic nature of the geometric properties of shipbuilding structural profiles results from the probabilistic nature of the corrosion wear (only uniformly distributed corrosion wear within each cross section’s dimension is considered). The problem is solved using the Taylor series expansion as a basic method in this work. The essence of the method is the linearization of a function $Y$ (in this case – any geometric property such as Area, Moment of Inertia, etc.) of random arguments $x_i$ (i.e. cross section’s dimensions) within the range of possible values of these arguments. It is applied in the vicinity of the mean values of the arguments. Assuming statistically independent arguments, the mean and variance of $Y$ are [12]):

$$ \bar{Y} = F(\bar{x}_i) $$

$$ D_Y = \sum_i \left( \frac{\partial F(\bar{x}_i)}{\partial x_i} \right)^2 D_i $$

where:

$ \bar{x}_i $ is the mean value of $x_i$.

$ D_i $ is the variance of $x_i$.

Consequently, the mean values of the geometric properties of structural profiles (Cross Section Area, Static Moments, Moments of Inertia, etc.) can be calculated with the existing methods and computer programs for their deterministic calculation, substituting the cross section’s dimensions with the corresponding mean values of $x_i$. The calculation of the variance of these geometric properties requires derivation of the first derivatives relatively to each cross section’s dimension $x_i$. After that these first derivatives should be calculated substituting in the equations the cross section’s dimensions with their mean values.

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Eqs. (1) and (2) represent the so-called first-order approximation. In his previous works ([5], [6], [8]) the author used second-order approximation. It was found that the difference between first-order and second-order approximation is negligible. In the extreme cases the difference barely reached 0.05-0.10%. This is the reason to use at present the first-order approximation, which substantially simplifies the calculations without jeopardizing the accuracy. The author also found that the above-mentioned geometric properties of shipbuilding structural profiles obey the Gaussian (normal) distribution. Strictly speaking, the normal distribution should be truncated because these geometric properties cannot be neither greater than their corresponding initial values nor decrease indefinitely during the ship's operational lifetime. Therefore, one should recalculate the ordinates of the normal distribution with boundaries \(-\infty, +\infty\) in order to obtain the ordinates of the truncated normal distribution. However, the author found [4] that for the corrosion wear reported in different publications, such as [2], [3], [9], [10], this change is so insignificant that it can be discarded.

The procedure for probabilistic calculation of the geometric properties contains the following phases:

1. The mean values and standard deviations of the corrosion wear are determined for the attached plate, web plate, and the bulb head/flange for each year of the ship's operational lifetime.
2. The mean values and standard deviations of all dimensions of the structural profile's cross section are determined for each year of the ship's lifetime.
3. The mean values of any geometric property are determined with eq. (1) for each year of the ship's lifetime.
4. The first derivatives of any geometric property relatively to each dimension of the structural profile's cross section are determined.
5. The variance/standard deviation of any geometric property is determined with eq. (2).
6. Knowing the mean values and standard deviations of any geometric property, its normal distribution function is built for any ship's.
7. The envelope of the normal distribution functions is built by putting together the probability density functions calculated for each year of the ship's lifetime (see Fig.1).

Let's continue further with the parameter \(Y\) presented in non-dimensional format:

\[
y = \frac{Y}{Y_{\text{nom}}}
\]

where \(Y_{\text{nom}}\) is the nominal value of \(Y\).

The probabilistic presentation of the geometric properties allows for calculation of the:

- Probability of the geometric property, \(y\), being between \(y_u\) and \(y_l\) at time \(T\), i.e.
  \[
P_y(y_u \leq y \leq y_l; T) = \int_{y_u}^{y_l} p_T(y) \, dy
\]
  where \(p_T(y)\) is the probability density function of given geometric property \(y\) at time \(T\)

- Probability of the geometric property, \(y\), being between any \(y_u\) and \(y_l\) over time interval \((T_u, T_f)\), i.e.
  \[
P_y(y_u \leq y \leq y_l; T_u \leq T \leq T_u) = \frac{1}{T_u - T_f} \left[ \int_{T_f}^{T_u} \left( \int_{y_u}^{y_l} p_T(y) \, dy \right) \, dT \right]
\]

\(Figs. 1 and 2 were initially drawn by Eng. N. Todorov from the Technical University, Varna (Bulgaria) and later completed by the author.\)
where:

- \( y_u \) is the upper boundary of \( y \)
- \( y_l \) is the lower boundary of \( y \)
- \( T_u \) is the upper boundary of the time \( T \)
- \( T_l \) is the lower boundary of the time \( T \)
- \( p_T(y) \) is the probability density function of \( y \) at time \( T \)

Eq. (5) represents a kind of "average" probability that \( y \) will not exceed given limits within a specified time-interval. It indicates the level of confidence one may have when assessing the acceptability of the deteriorated stiffener's geometric properties.

The nominator's geometric interpretation of eq. (5) is the volume below the envelope within given boundaries \( T_u \), \( T_l \) and \( y_u \), \( y_l \) as shown in Fig. 2. It may be called volume of a "deformed" parallelepiped. Thus, the "average" probability \( P ( y_l \leq y \leq y_u ; T_l \leq T \leq T_u ) \) is equal to the volume of the "deformed" parallelepiped divided by \( T_u - T_l \).

As to the geometric interpretation of eq. (4), it is equal to the area below the probability density function of \( y \) for time \( T \) within given boundaries \( y_u \) and \( y_l \) (see the crosshatched area in Fig. 2).

II. ANALYZED GEOMETRIC PROPERTIES

Altogether twenty six geometric properties of shipbuilding structural profiles (bulb plates, inverted angles/L-profiles, channels, symmetric or asymmetric built-up T-bars, etc.) were analyzed.

As to the type of geometric properties, all traditionally used properties such as cross section area, moments of inertia, etc. were analyzed, including the Section Modulus for asymmetric bending introduced by the author [4].

III. EQUATIONS FOR THE GEOMETRIC PROPERTIES AND THEIR FIRST DERIVATIVES

These equations were derived for all shipbuilding structural profiles [4]. To have some idea about the scope of work, the results for the geometric properties of built-up T-bars (Fig. 3) relatively to the original axes X and Y are given in Appendix A. Once the geometric properties relatively to the original axes X and Y are determined, one can calculate all other geometric properties such as coordinates of the centroid, Moments of Inertia, etc [7]. Then, the derivatives of each...
geometric property are derived for each type of structural profiles (see [4]). After that one can calculate the probabilities $P ( y_l \leq y \leq y_u ; T_l \leq T \leq T_u )$ and $P ( y_l \leq y \leq y_u ; T )$ following the procedure given in the INTRODUCTION.

For the sake of brevity, the results for only seven built-up T-bar’s geometric properties are given here as an example:

- Cross section area, $A_S$, of the built-up T-bar
- Total cross section area, $A$, of the built-up T-bar and the attached plate
- Centroidal Moment of Inertia, $I_{X1}$, relatively to the horizontal centroidal axis
- Section Modulus, $SM_{sym}$, for sym. bending
- Section Modulus, $SM_{asym}$, for asym. bending
- Thickness, $t_w$, of the web plate
- Thickness, $t_2$, of the flange

**IV. CORROSION WEAR**

The purpose of this work is to determine the effect of the corrosion wear on the geometric properties of shipbuilding structural profiles. The proposed methods do not depend on the type of input data for the corrosion wear. Therefore, methods for corrosion wear prediction are not discussed here.

To ensure continuity of the built-up T-bar’s cross section when it shrinks due to overall corrosion, the assumed new cross section shape is shown in Fig. 4

Under the term “time T” the difference between the ship’s age and the longevity of the coating is meant, i.e.

$$T = T_s - T_c$$  \hspace{1cm} (6)$$

where:

- $T_s$ is the ship’s age
- $T_c$ is the longevity of the corrosion protection

For any cross section dimension, $x$, one can write:

$$x_T = x_o - \delta_{x,T}$$  \hspace{1cm} (7)$$

$$D_{x,T} = D_{x,o} + D_{\delta,T}$$  \hspace{1cm} (8)$$
where:

\( x_T \) is the value of \( x \) at time \( T \)
\( x_0 \) is the initial value of \( x \)
\( \delta_{x,T} \) is the corrosion wear of \( x \) at time \( T \)
\( \bar{x}_T \) is the mean value of \( x \) at time \( T \)
\( \bar{x}_o \) is the mean of the initial value of \( x \). It is assumed here that it is equal to the mean of the nominal value of \( x \). Usually, it deviates from the nominal values given in the specifications for shipbuilding structural profiles up to 1-2% (see [5], [6], [8]).
\( \delta_{x,T} \) is the mean value of the corrosion wear of \( x \) at time \( T \)
\( D_{x,T} \) is the variance of \( x \) at time \( T \)
\( D_{x,o} \) is the variance of the initial value of \( x \). It is assumed here as equal to the variance of the nominal value of \( x \)
\( D_{\delta,T} \) is the variance of the corrosion wear of \( x \) at time \( T \)

For the example to follow, a constant average corrosion wear is assumed throughout the ship’s lifetime after failure of the coating. Then, any parameter \( x \) at time \( T \) can be calculated with the formula:

\[
\bar{x}_T = x_0 - \delta_{x,T} = x_0 - T \delta_{x,a}
\]

(9)

where \( \delta_{x,a} \) is the average corrosion wear per unit time of any parameter \( x \). Consequently, the mean and the variance of \( x \) will be:

\[
\bar{x}_T = \bar{x}_o - T \delta_{x,a}
\]

\[
D_{x,T} = D_{x,o} + T^2 D_{\delta,a}
\]

(10)

where \( D_{\delta,a} \) is the variance of the average corrosion wear of \( x \) per unit time

V. NUMERICAL EXAMPLE FOR ASYMMETRIC BUILT-UP T-BAR

The cross section’s dimensions of the built-up asymmetric T-bar are given in Table 1. The nominal values of some of its geometric properties are given in Table 2. The mean values and variances of the corrosion wear used in this example are given in Table 3.

| Table 1. Nominal values of the cross section’s dimensions of the asymmetric built-up T-bar (see Fig. 4) |
|---|---|---|
| Built-up T-bar | H | cm | 47.00 |
| | B | cm | 15.00 |
| | C | cm | 10.00 |
| | \( t_2 \) | cm | 2.00 |
| | \( t_w \) | cm | 1.10 |
| Attached plate | \( t_\theta \) (thickness) | cm | 1.10 |
| | \( l_p \) (width) | cm | 80.00 |
| | \( \theta \) (angle relatively to axis X) | degrees | 15.00 |
### Table 2. Nominal values of some of the geometric properties of the built-up asymmetric T-bar

<table>
<thead>
<tr>
<th>Parameter</th>
<th>dimension</th>
<th>notation</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Cross Section Area</td>
<td>cm²</td>
<td>A</td>
<td>167.50</td>
</tr>
<tr>
<td>Cross Section Area of the built-up T-bar alone</td>
<td>cm²</td>
<td>Aₛ</td>
<td>79.50</td>
</tr>
<tr>
<td>Ordinate of the Centroid relatively to original axis X</td>
<td>cm</td>
<td>eₓ</td>
<td>14.53</td>
</tr>
<tr>
<td>Abscissa of the Centroid relatively to original axis Y</td>
<td>cm</td>
<td>eᵧ</td>
<td>0.52</td>
</tr>
<tr>
<td>Moment of Inertia relatively to Horizontal Centroidal axis</td>
<td>cm⁴</td>
<td>Iₓ₁</td>
<td>64725.07</td>
</tr>
<tr>
<td>Moment of Inertia relatively to Vertical Centroidal axis</td>
<td>cm⁴</td>
<td>Iᵧ₁</td>
<td>44501.03</td>
</tr>
<tr>
<td>Angle of the Principle Axes X₂ relatively to Horizontal Centroidal axis *</td>
<td>degrees</td>
<td>αₓ₂</td>
<td>-26.97</td>
</tr>
<tr>
<td>Moment of Inertia relatively to Principal axis X₂</td>
<td>cm⁴</td>
<td>Iₓ₂ = Iₘₘₙₐₓ</td>
<td>71790.28</td>
</tr>
<tr>
<td>Moment of Inertia relatively to Principal axis Y₂</td>
<td>cm⁴</td>
<td>Iᵧ₂ = Iₘᵢₙₐₓ</td>
<td>37435.82</td>
</tr>
<tr>
<td>Section Modulus for symmetric bending</td>
<td>cm³</td>
<td>SMₛₖᵢₐₘ</td>
<td>1993.47</td>
</tr>
<tr>
<td>Angle between the &quot;Elastic&quot; N.A. for sym. and asymmetric bending *</td>
<td>degrees</td>
<td>ψ</td>
<td>17.33</td>
</tr>
<tr>
<td>Section Modulus for asymmetric bending</td>
<td>cm³</td>
<td>SMₛₐₘₖₐₘ</td>
<td>1766.29</td>
</tr>
</tbody>
</table>

* Sign "+" of the angle means rotation clock wise relatively to the axis for comparison

Two general cases were considered:

1. The lower boundaries for all geometric properties are assumed the same. These calculations are denoted with "variant I".
2. The reduction of the cross section's dimensions due to corrosion is first determined. Then, the corresponding new value of any geometric property is found and used as its lower boundary. These calculations are denoted with "variant II".

Variant II.a refers to case when the permissible reduction of the thickness of the flange and web plate is 25% while variant II.b refers to case when this permissible reduction is assumed as equal to 10%

For the built-up T-bar under consideration the results for the probabilities $P \left( yₙ ≤ y ≤ yₚ ; T \right)$ for the two variants are given in Figs. 5, 6 and 7. Table 4 contains the input data used for their calculations.

The assumed coating’s longevity is 2 years while the assumed ship’s operational lifetime is 25 years. The assumed acceptable risk for not meeting the Renewal Criteria requirements is assumed as equal to 10% (i.e. the acceptable probability of meeting these requirements is 90% as shown in Table 4). Table 5 contains results for the probabilities $P \left( yₙ \leq y \leq yₚ ; Tₙ \leq T \leq Tₚ \right)$ calculated for the input given in Table 4.

### Table 3. Mean values and Variances of the corrosion wear $δₐₚ$ used as input for the calculations

<table>
<thead>
<tr>
<th>Flange</th>
<th>$\bar{δ}_{ₓₐₚ}$ [mm/year]</th>
<th>$D_{ₓₐₚ}$ [mm/year]²</th>
<th>$\bar{δ}_{hor,ₐₚ}$ [mm/year]</th>
<th>$D_{hor,ₐₚ}$ [mm/year]²</th>
<th>Web plate</th>
<th>$\bar{δ}_{wₚₐₚ}$ [mm/year]</th>
<th>$D_{wₚₐₚ}$ [mm/year]²</th>
<th>Attached plate</th>
<th>$\bar{δ}_{pₚₐₚ}$ [mm/year]</th>
<th>$D_{pₚₐₚ}$ [mm/year]²</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flange</td>
<td>0.100</td>
<td>0.0016</td>
<td>0.100</td>
<td>0.0016</td>
<td></td>
<td>0.100</td>
<td>0.000096</td>
<td></td>
<td>0.107</td>
<td>0.00096</td>
</tr>
</tbody>
</table>
Table 4. Input data for calculation of $P( y_l \leq y \leq y_u ; T )$ and $P( y_l \leq y \leq y_u ; T_l \leq T \leq T_u )$

<table>
<thead>
<tr>
<th>variant</th>
<th>same boundaries for all $y$ and $t_w$</th>
<th>$y_u$</th>
<th>$y_l$</th>
<th>$T_u$</th>
<th>$T_l$</th>
<th>$T$</th>
<th>Percentage thickness loss along the cross section</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>$y_l$ follows from the thickness loss</td>
<td>$y_l$</td>
<td>$A_S$</td>
<td>$A$</td>
<td>$I_{X1}$</td>
<td>SM$_{sym}$</td>
<td>SM$_{asym}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.750</td>
<td>0.750</td>
<td>0.748</td>
<td>0.753</td>
<td>0.753</td>
<td>10.00</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.900</td>
<td>0.900</td>
<td>0.898</td>
<td>0.901</td>
<td>0.901</td>
<td></td>
</tr>
</tbody>
</table>

Assumed acceptable probability [%] of meeting the requirements of the Renewal Criteria 90.00

Table 5. “Average” probabilities $P( y_l \leq y \leq y_u ; T_l \leq T \leq T_u )$ for the asymmetric built-up T-bar

<table>
<thead>
<tr>
<th>Variant</th>
<th>Parameter</th>
<th>$t_w$</th>
<th>$t_2$</th>
<th>$A_S$</th>
<th>$A$</th>
<th>$I_{X1}$</th>
<th>SM$_{sym}$</th>
<th>SM$_{asym}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td></td>
<td>0.83053</td>
<td>0.99996</td>
<td>0.98029</td>
<td>0.98825</td>
<td>0.99929</td>
<td>0.99993</td>
<td>0.99989</td>
</tr>
<tr>
<td>II.a</td>
<td></td>
<td>0.83053</td>
<td>0.99996</td>
<td>0.98005</td>
<td>0.98816</td>
<td>0.99943</td>
<td>0.99990</td>
<td>0.99985</td>
</tr>
<tr>
<td>II.b</td>
<td></td>
<td>0.06933</td>
<td>0.44709</td>
<td>0.09771</td>
<td>0.03206</td>
<td>0.08356</td>
<td>0.13750</td>
<td>0.13498</td>
</tr>
</tbody>
</table>

Fig. 5. $P(y_l \leq y \leq y_u ; T )$ for $A_S$, $A$, $I_{X1}$, SM$_{sym}$, SM$_{asym}$, $t_2$ and $t_w$ for the asymmetric built-up T-bar in variant I
Fig. 6. \( P( y_l \leq y \leq y_u ; T) \) for As, A, I_{X1}, S_{Msym}, S_{Masym}, t_2 and \( t_a \) for the asym. built-up T-bar in variant II.a.

Fig. 7. \( P( y_l \leq y \leq y_u ; T) \) for As, A, I_{X1}, S_{Msym}, S_{Masym}, t_2 and \( t_a \) for the asym. built-up T-bar in variant II.b.
VII. ANOTHER APPROXIMATE METHOD FOR PROBABILISTIC PRESENTATION OF THE GEOMETRIC PROPERTIES

The author proposed and tested another simple approximate method for probabilistic presentation of the geometric properties of shipbuilding structural profiles. It is based on the same principle used for calculation of the mean value of the corresponding geometric property, $Y$, in Taylor series expansion given in eq. (1). Let’s calculate $Y$ with the same equation but with the following $x_i$:

$$Y = F\left(x_i = \bar{x} + \sigma_i\right)$$

where $\bar{x}_i$ is the mean value of $x_i$ and $\sigma_i = \sqrt{D_i}$ is the standard deviation of any cross section’s dimension $x_i$.

Then, the standard deviation of the function $Y$ (in this case any geometric property) is determined with the equation:

$$\sigma_Y = F\left(x_i = \bar{x} + \sigma_i\right) - F\left(x_i = \bar{x}\right)$$

(12)

Note: the mean values of the geometric properties are the same in both methods.

The numerical verification was performed against results obtained with Taylor series expansion method for a bulb plate with cross section’s dimensions given in Table 6.

Table 6. Nominal values of the cross section’s dimensions of the bulb plate

<table>
<thead>
<tr>
<th>Parameter</th>
<th>dimensions</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>bulb plate’s height</td>
<td>cm</td>
<td>25.00</td>
</tr>
<tr>
<td>bulb head’s width</td>
<td>cm</td>
<td>4.50</td>
</tr>
<tr>
<td>web plate’s thickness</td>
<td>cm</td>
<td>1.20</td>
</tr>
<tr>
<td>angle of the inclined edge</td>
<td>degrees</td>
<td>30</td>
</tr>
<tr>
<td>attached plate’s thickness</td>
<td>cm</td>
<td>1.20</td>
</tr>
<tr>
<td>attached plate’s width</td>
<td>cm</td>
<td>70.00</td>
</tr>
<tr>
<td>Angle of the attached plate</td>
<td>degrees</td>
<td>15</td>
</tr>
</tbody>
</table>

The standard deviations of five geometric properties ($A_S$, $A$, $I_{X1}$, $SM_{sym}$ and $SM_{asym}$) are calculated by both methods and given in Table 7. One can notice that the standard deviations obtained by the proposed method are greater than those obtained by Taylor method. The effect of the differences between the standard deviations obtained by the two methods on the probabilities $P(y_l \leq y \leq y_u; T)$ was numerically evaluated and graphically illustrated in Figs. 8 and 9. Only the probabilities $P(y_l \leq y \leq y_u; T)$ for the Cross Section Area of the bulb plate, $A_S$, and the Centroidal Moment of Inertia relatively to the horizontal Neutral Axes, $I_{X1}$, are shown in these figures: The probability $P(y_l \leq y \leq y_u; T)$ for $I_{X1}$ is selected as an example because the difference between its standard deviations obtained by the two methods is maximal related to all other standard deviations of the geometric properties. At the same time the difference between the probabilities $P(y_l \leq y \leq y_u; T)$ for the Cross Section Area of the bulb plate obtained by the two methods is minimal. Thus, the whole range of differences is covered, which facilitates drawing conclusions for the applicability of the approximate method.

As to the data for the corrosion wear used in the calculations for the bulb plate, they are the same as those given in Table 3.

Table 7. Standard deviations of the geometric properties of the bulb plate

<table>
<thead>
<tr>
<th>Parameter</th>
<th>year</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_S$</td>
<td></td>
</tr>
<tr>
<td>Proposed method</td>
<td>0.9585</td>
</tr>
<tr>
<td>Taylor series</td>
<td>0.6547</td>
</tr>
<tr>
<td>$A$</td>
<td></td>
</tr>
<tr>
<td>Proposed method</td>
<td>2.4285</td>
</tr>
<tr>
<td>Taylor series</td>
<td>1.6092</td>
</tr>
<tr>
<td>$I_{X1}$</td>
<td></td>
</tr>
<tr>
<td>Proposed method</td>
<td>317.23</td>
</tr>
<tr>
<td>Taylor series</td>
<td>161.06</td>
</tr>
<tr>
<td>$SM_{sym}$</td>
<td></td>
</tr>
<tr>
<td>Proposed method</td>
<td>12.80</td>
</tr>
<tr>
<td>Taylor series</td>
<td>6.59</td>
</tr>
<tr>
<td>$SM_{asym}$</td>
<td></td>
</tr>
<tr>
<td>Proposed method</td>
<td>13.17</td>
</tr>
<tr>
<td>Taylor series</td>
<td>7.99</td>
</tr>
</tbody>
</table>
Fig. 8 Comparison between the results obtained for the probability $P(y_l \leq y \leq y_u ; T)$ with Taylor series method and the proposed method. The permissible reduction of the web plate's thickness is assumed = 25% while the corresponding permissible reduction of $A_S$ is = 21.90% and that of $I_{X1}$ is = 22.20%.

Fig. 9 Comparison between the results obtained for the probability $P(y_l \leq y \leq y_u ; T)$ with Taylor series method and the proposed method. The permissible reduction of the web plate's thickness is assumed = 10% while the corresponding permissible reduction of $A_S$ is = 8.80% and that of $I_{X1}$ is = 9.00%.

Based on Figs. 8 and 9, one can draw the following conclusions:

1. The difference between calculated probabilities with the two methods is greater for geometric properties of “higher” rank (e.g. $I_{X1}$) than for geometric properties of “lower” rank (e.g. $A_S$).
2. The maximal difference between the two probabilities $P(y_l \leq y \leq y_u ; T)$ is around 15%. For case when the assumed permissible reduction of the web plate’s thickness is 25% (on average, this is the permissible reduction of the web plate’s thickness in [1] after which replacement of the stiffener/girder is required). For this particular case (when renewal of the stiffener/girder is required), the approximate method produces more conservative results than those obtained with Taylor series method.
3. For case when the assumed permissible reduction of the web plate’s thickness is relatively small (e.g. 10% as shown in Fig. 10), the proposed method produces more conservative results for the first part of the ship’s lifetime while for the second part of its lifetime the trend is reverse.

Strictly speaking, the proposed method is mathematically incorrect. However, the question is **HOW MUCH INCORRECT?** To answer this question and to find out the limitations of the method, the author performed the following analysis:

The corrosion wear of ship structures affects the thickness of the steel plates, web plates, flanges of built-up T-bars or bulb head of rolled profiles. The width of the steel plates is not affected. The height and the flanges'/bulb heads’ width is insignificantly changed. Consequently, one can conclude that:

1. The Cross section Area of the plates and built-up T-bars is linear function of the thickness.
2. The Cross Section Area of rolled stiffeners is very close to linear function of the thickness and can be treated as linear one with confidence. The geometric properties of “higher rank” geometric properties such as Static Moments, Moments of Inertia, etc. are also very close to linear functions of the area (hence, the thickness). The reason for it is that the distance from the centroid of each structural component to given axis for comparison does not change much due to corrosion wear. The change of this distance is few orders of magnitude smaller that that of the cross section area due to corrosion wear. This is a known fact and one can rely on it to assume that the geometric properties used for calculation of the elastic strength of ship structures are either linear (e.g. Cross Section Area) or very close to linear functions of the thickness.

Let’s represent any non-dimensional geometric property, \( y \), in the most general form:

\[
y = a_1x_1 + a_2x_2 + \ldots + a_kx_k = \sum_{i=1}^{k} a_i x_i
\]

(13)

where the geometric interpretation of any coefficient \( a_i \) is the tangent of the angle between the tangent line to \( y \) and axis \( x_i \). The mathematical interpretation of \( a_i \) is the first derivative of \( y \) relatively to each \( x_i \). If eq. (12) is applied to eq. (13), one can derive the following expression for the standard deviation of \( y \):

\[
\sigma_{y, \text{proposal}} = \sum_{i=1}^{k} a_i (\bar{x}_i + \sigma_i) - \sum_{i=1}^{k} a_i (\bar{x}_i) = \sum_{i=1}^{k} a_i \sigma_i
\]

(14)

where \( \sigma_{y, \text{proposal}} = \sqrt{D_{y, \text{proposal}}} \) is the standard deviation of \( y \) derived with the proposed method.

If eq. (2), which is a part of Taylor series method is applied to eq. (13), the standard deviation of \( y \) will be:

\[
\sigma_{y, \text{Taylor}} = \sqrt{\sum_{i=1}^{k} a_i^2 \sigma_i^2}
\]

(15)

where \( \sigma_{y, \text{Taylor}} \) is the standard deviation of \( y \) derived with Taylor series method.

Eq. (15) can be rewritten the following way:

\[
\sigma_{y, \text{Taylor}} = \sqrt{\left(\sum_{i=1}^{k} a_i \sigma_i\right)^2 - 2\sum_{i=j} a_i a_j \sigma_i \sigma_j}
\]

(16)
The two standard deviations obtained by the proposed and Taylor series method will be equal if the second term under the square root in eq. (16) is equal to zero, i.e.

\[
\sigma_{y, \text{proposl}} = \begin{cases} 
\sigma_{y, \text{Taylor}} & \text{if } \sum_{i \neq j} a_i a_j \sigma_i \sigma_j = 0 \\
\sigma_{y, \text{Taylor}} > & \text{if } \sum_{i \neq j} a_i a_j \sigma_i \sigma_j \neq 0 
\end{cases}
\] (17)

The term \( \sum_{i \neq j} a_i a_j \sigma_i \sigma_j \) is practically never equal to zero, i.e. always \( \sigma_{y, \text{Taylor}} \neq \sigma_{y, \text{proposl}} \) although the difference between the two standard deviations is not big (see Table 7) due to following reasons:

- The derivatives of \( y \) relatively to \( x_i \) and \( x_j \), \( a_i \) and \( a_j \), are of different signs [4]. Consequently, the products \( 2a_i a_j \sigma_i \sigma_j \) may balance each other.
- In addition, what is left from the term \( \sum_{i \neq j} a_i a_j \sigma_i \sigma_j \) does not have strong effect on the difference between the standard deviations because of the rooting procedure.

In other words, the probability density functions of \( y \) calculated by the two methods always have the form shown in Fig. 10. The corresponding cumulative distribution functions are shown in Fig. 11.

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**Fig. 10** Probability density functions (p.d.f.) obtained by the proposed and Taylor series method. The example is given for the Centroidal Moment of Inertia relatively to the horizontal Neutral Axis of a bulb plate. The ship’s age is 25 years.
Taylor Series Applied for Probabilistic Presentation of the Geometric Properties of Structural Profiles

Fig. 11 Cumulative distribution functions (c.d.f.) obtained by the proposed and Taylor series method. The example is given for the Centroidal Moment of Inertia relatively to the horizontal Neutral Axis of a bulb plate. The ship’s age is 25 years.

Fig. 11 is very convenient to explain the differences between the probabilities \( P( y \leq y_1 \leq y_u : T ) \) obtained by the two methods. Let’s assume that for ship’s age = 25 years, the permissible reduction of \( I_{X1} \) is \( (1-\alpha)I_{X1,\text{nom}} \). Then, the probability that given \( y \) (in this case \( I_{X1} \)) will be above the assumed permissible limit can be calculated with the equation:

\[
P( y \geq 1-\alpha ) = 1 - F( y = \alpha )
\]

where:
- \( P( y \geq 1-\alpha ) \) is the probability that \( I_{X1} \) will be greater than \( (1-\alpha)I_{X1,\text{nom}} \)
- \( F( y = \alpha ) \) is the cumulative distribution function of \( I_{X1} \) calculated for \( y = \alpha \). There are three possible values for \( \alpha \) (see Fig. 11), for which the ratio between the probabilities \( P( y \geq 1-\alpha ) \) derived by the proposed and Taylor series method is:

- \( 1-\alpha_1 < y \) then \( P_{\text{proposed}} \left( y \geq 1-\alpha_1 \right) < P_{\text{Taylor}} \left( y \geq 1-\alpha_1 \right) \)
- \( 1-\alpha_2 = y \) then \( P_{\text{proposed}} \left( y \geq 1-\alpha_2 \right) = P_{\text{Taylor}} \left( y \geq 1-\alpha_2 \right) \)
- \( 1-\alpha_3 > y \) then \( P_{\text{proposed}} \left( y \geq 1-\alpha_3 \right) > P_{\text{Taylor}} \left( y \geq 1-\alpha_3 \right) \)

In other words, when the permissible reduction of \( y \) is greater than the reduction of the mean value of \( y \), the proposed method produces more conservative results than Taylor series method. The two methods produce the same or almost the same results when the permissible reduction is equal or close to the reduction of the mean value of \( y \). When the permissible reduction is smaller than the reduction of the mean value of \( y \), the proposed method produces results in the non-conservative side.

For problems of this type that contain so many uncertainties, difference of the order quoted above might be considered acceptable. The proposed approximate method becomes even more attractive while comparing its simplicity with the ambiguous analytical equations derived for all shipbuilding structural profiles in [4]. Once confirmed, the new approximate method could become very efficient tool in application of the Time-dependent approach to ship structural design, maintenance and repair because of its simplicity, sufficient accuracy and ease in the application.
VIII  CONCLUSIONS

1. An analytical method for presentation of the geometric properties used in assessing the elastic bending strength of all types of shipbuilding structural profiles is developed based on Taylor series expansion. The solution is in closed format, which allows for quick explicit estimates of each input parameter on any geometric property. Although the derivation of the equations is tedious work, once done, tested and verified, the programming of the calculations would not be very difficult. It can be done even with such popular and accessible computer programs as Microsoft EXCEL (the author obtained all numerical results using this computer program).

2. A new approximate method for probabilistic presentation of the geometric properties of shipbuilding structural profiles is proposed that allows for fast and simple calculations with sufficient accuracy. The existing computer programs for deterministic calculation of the geometric properties can be used for this purpose, which in itself can save a lot of efforts and time. One should calculate the mean values and standard deviations of each cross section’ dimension and use them as input for calculation of the mean and standard deviation of any geometric property. It was shown by the author in his previous works that the geometric properties of shipbuilding structural profiles obey the normal (Gaussian) distribution. Therefore, these two numerical characteristics are sufficient to represent any of the geometric properties in probabilistic terms. After that is easy to calculate either the probability \( P \left( y_l \leq y \leq y_u ; T \right) \) or \( P \left( y_l \leq y \leq y_u ; T_l \leq T \leq T_u \right) \).

3. The existing Renewal Criteria for stiffeners and girders control only the geometric properties. This practice is based on the past experience in dealing with the strength of aging ships. However, the quest for increasing ship safety will soon lead to a shift in this practice towards strength analysis of aging ships. Then, the proposed methods for probabilistic presentation of the geometric properties of shipbuilding structural profiles will become one of the key components in the strength calculations together with the probabilistic presentation of the loads and the mechanical properties of the material.

4. The presented methods can be applied in the design stage. It allows for comparison of the effectiveness of the structural profiles using as a criteria the probability that different structural profiles with the same or close SM, Moments of Inertia, etc will meet the requirements of the Renewal Criteria throughout the presumed ship's lifetime. For the "as-built" ship, they may have the same geometric properties, but with time the probability of meeting the requirements of the Renewal Criteria may differ substantially.

5. The methods can also be applied during the ship's operation, repair and maintenance. Performing calculations as described in the report, the interested parties will obtain valuable information about the probability that given structural profile will meet the requirements of the Renewal Criteria until the next survey.

6. It is possible with the developed methods to analyze the existing Renewal Criteria for a variety of structural components, locations, ship's types and size in order to reveal the already implemented risk in these Criteria.

ACKNOWLEDGEMENT

The author would like to thank the management of ABS, Technology Division, for its support of this work, especially Dr. John Spencer. Thanks are due to Prof. L M Belenky, Albert Ku, Raymond Ng, Alfred Tunik, Ge Wang, John Parente, Ah Kuan Seah, Yury Raskin who contributed to the report through professional criticism.
Appendix A  Equations for asymmetric built-up T-bars

The structural profile is divided in two parts by a plane passing vertically through the mean thickness of the web plate (see Fig. 4). The properties of the part on the right of axis Y are marked with “r” and those to the left of Y are marked with “l”. The properties of the stiffener are marked with “s” and those of the attached plate are marked with “p”.

Cross section Area

\[ A_S = A_R + A_I \]
\[ A_R = c t_2 + \frac{t_w}{2} (h-t_2) \]
\[ A_I = (b-c) t_2 + \frac{t_w}{2} (h-t_2) \]  
(A1)

\[ \frac{\partial A_S}{\partial x_i} = \frac{\partial A_R}{\partial x_i} + \frac{\partial A_I}{\partial x_i} \]  
(A2)

The derivatives that are different from zero are:

\[ \frac{\partial A_R}{\partial t_w} = c - \frac{t_w}{2} \]
\[ \frac{\partial A_I}{\partial t_w} = b - c - \frac{t_w}{2} \]  
\[ \frac{\partial A_R}{\partial t_2} = \frac{t_w}{2} \]
\[ \frac{\partial A_I}{\partial t_2} = \frac{t_w}{2} \]
\[ \frac{\partial A_R}{\partial b} = - \frac{\partial A_I}{\partial c} = t_2 \]
\[ \frac{\partial A_R}{\partial c} = - \frac{\partial A_I}{\partial b} \]  
(A3)

\[ \frac{\partial A_R}{\partial t_2} = c - \frac{t_w}{2} \]
\[ \frac{\partial A_I}{\partial t_2} = b - c - \frac{t_w}{2} \]
\[ \frac{\partial A_R}{\partial t_w} = \frac{h-t_2}{2} \]
\[ \frac{\partial A_I}{\partial t_w} = \frac{h-t_2}{2} \]  
(A4)

Static Moment relatively to axis X

\[ S_{X,S} = S_{X,R} + S_{X,I} \]
\[ S_{X,R} = \frac{1}{2} \left[ (2h-t_2) c t_2 + \frac{t_w}{2} (h-t_2)^2 \right] \]
\[ S_{X,I} = \frac{1}{2} \left[ (2h-t_2)(b-c) t_2 + \frac{t_w}{2} (h-t_2)^2 \right] \]  
(A5)

(A6)
\[ \frac{\partial S_{X,S}}{\partial x_i} = \frac{\partial S_{X,r}}{\partial x_i} + \frac{\partial S_{X,l}}{\partial x_i} \]  

(A7)

The derivatives that are different from zero are:

\[ \frac{\partial S_{X,r}}{\partial h} = A_r, \quad \frac{\partial S_{X,l}}{\partial h} = A_l, \quad \frac{\partial S_{X,r}}{\partial b} = \frac{\partial S_{X,l}}{\partial b} = \frac{t_2}{2} \quad \frac{\partial S_{X,r}}{\partial c} = \frac{\partial S_{X,l}}{\partial c} = \frac{t_2}{2} \]  

\[ \frac{\partial S_{X,r}}{\partial t_2} = h c - A_r, \quad \frac{\partial S_{X,l}}{\partial t_2} = h (b - c) - A_l \]  

(A8)

Static Moment relatively to axis \( Y \)

\[ S_{Y,S} = S_{Y,r} - S_{Y,l} \]  

(A10)

\[ S_{Y,r} = \frac{1}{2} \left[ c^2 t_2 + \frac{t_2^2}{4} (h - t_2) \right] \quad S_{Y,l} = \frac{1}{2} \left[ (b - c)^2 t_2 + \frac{t_2^2}{4} (h - t_2) \right] \]  

(A11)

\[ \frac{\partial S_{Y,S}}{\partial x_i} = \frac{\partial S_{Y,r}}{\partial x_i} - \frac{\partial S_{Y,l}}{\partial x_i} \]  

(A12)

The derivatives that are different from zero are:

\[ \frac{\partial S_{Y,r}}{\partial h} = \frac{\partial S_{Y,l}}{\partial h} = \frac{t_2^2}{8}, \quad \frac{\partial S_{Y,r}}{\partial b} = (b - c) t_2, \quad \frac{\partial S_{Y,l}}{\partial c} = c t_2 \quad \frac{\partial S_{Y,r}}{\partial t_2} = -\frac{t_2^2}{4} \quad \frac{\partial S_{Y,l}}{\partial t_2} = \frac{t_2^2}{4} (h - t_2) \]  

(A13)

Moment of Inertia relatively to axis \( X \)

\[ I_{X,S} = I_{X,r} + I_{X,l} \]  

(A15)

\[ I_{X,r} = \frac{t_2 h^3}{6} + t_2 \left( c - \frac{t_2}{2} \right) \left( \frac{t_2^2}{6} + \frac{t_2}{3} (h - t_2) \right) \quad I_{X,l} = \frac{t_2 h^3}{6} + t_2 \left( b - c - \frac{t_2}{2} \right) \left( \frac{t_2^2}{6} + \frac{t_2}{3} (h - t_2) \right) \]  

(A16)

\[ \frac{\partial I_{X,S}}{\partial x_i} = \frac{\partial I_{X,r}}{\partial x_i} + \frac{\partial I_{X,l}}{\partial x_i} \]  

(A17)

The derivatives that are different from zero are:

\[ \frac{\partial I_{X,r}}{\partial h} = 2 S_{X,r} \quad \frac{\partial I_{X,l}}{\partial h} = 2 S_{X,l} \]  

(A18)

\[ \frac{\partial I_{X,r}}{\partial b} = \frac{\partial I_{X,l}}{\partial c} = \frac{t_2}{2} \left[ h^2 - t_2 \left( h - \frac{t_2}{3} \right) \right] \]  

(A19)

\[ \frac{\partial I_{X,r}}{\partial t_2} = c h^2 - 2 S_{X,r} \quad \frac{\partial I_{X,l}}{\partial t_2} = (b - c) h^2 - 2 S_{X,l} \]  

(A20)

\[ \frac{\partial I_{X,r}}{\partial t_w} = \frac{\partial I_{X,l}}{\partial t_w} = \frac{1}{2} \left( \frac{h^3}{3} - \frac{\partial I_{X,r}}{\partial c} \right) \]  

(A21)

Moment of Inertia relatively to axis \( Y \)

\[ I_{Y,S} = I_{Y,r} + I_{Y,l} \]  

(A22)
\[ I_{Y,r} = \frac{1}{3} \left[ t_2 \left( \frac{c^3}{8} + \frac{t_2^3}{8} (h-t_2) \right) \right] \]  
\[ I_{Y,l} = \frac{1}{3} \left[ t_2 (b-c)^3 + \frac{t_2^3}{8} (h-t_2) \right] \]  
(A23)

\[ \frac{\partial I_{Y,S}}{\partial x_j} = \frac{\partial I_{Y,r}}{\partial x_i} + \frac{\partial I_{Y,l}}{\partial x_i} \]  
(A24)

The derivatives that are different from zero are:

\[ \frac{\partial I_{Y,r}}{\partial h} = \frac{\partial I_{Y,l}}{\partial h} = \frac{t_2^3}{24} \]  
\[ \frac{\partial I_{Y,l}}{\partial b} = \frac{\partial S_{Y,r}}{\partial b} t_2 \]  
\[ \frac{\partial I_{Y,r}}{\partial c} = \frac{\partial S_{Y,r}}{\partial c} t_2 \]  
\[ \frac{\partial I_{Y,l}}{\partial c} = -\frac{\partial I_{Y,l}}{\partial b} \]  
(A25)

\[ \frac{\partial I_{Y,r}}{\partial t_2} = \frac{1}{3} \left( c^3 - \frac{t_2^3}{8} \right) \]  
\[ \frac{\partial I_{Y,l}}{\partial t_2} = \frac{1}{3} \left( (b-c)^3 - \frac{t_2^3}{8} \right) \]  
(A26)

\[ \frac{\partial I_{Y,r}}{\partial t_w} = \frac{\partial I_{Y,l}}{\partial t_w} = \frac{\partial S_{Y,r}}{\partial h} (h-t_2) \]  
(A27)

Product of Inertia relatively to axes X and Y

\[ I_{XY,S} = I_{XY,r} - I_{XY,l} \]  
(A28)

\[ I_{XY,r} = \frac{1}{2} \left[ t_2 c^2 \left( h - \frac{t_2}{2} \right) + \frac{t_2^3}{8} (h-t_2)^2 \right] \]  
\[ I_{XY,l} = \frac{1}{2} \left[ t_2 (b-c)^2 \left( h - \frac{t_2}{2} \right) + \frac{t_2^3}{8} (h-t_2)^2 \right] \]  
(A29)

\[ \frac{\partial I_{XY,S}}{\partial x_i} = \frac{\partial I_{XY,r}}{\partial x_i} - \frac{\partial I_{XY,l}}{\partial x_i} \]  
(A30)

The derivatives that are different from zero are:

\[ \frac{\partial I_{XY,r}}{\partial h} = S_{Y,r} \]  
\[ \frac{\partial I_{XY,l}}{\partial h} = S_{Y,l} \]  
\[ \frac{\partial I_{XY,l}}{\partial b} = \frac{\partial S_{Y,r}}{\partial b} \left( h - \frac{t_2}{2} \right) \]  
\[ \frac{\partial I_{XY,l}}{\partial c} = -\frac{\partial I_{XY,l}}{\partial b} \]  
(A31)

\[ \frac{\partial I_{XY,r}}{\partial t_2} = \frac{h c^2}{2} - S_{Y,r} \]  
\[ \frac{\partial I_{XY,l}}{\partial t_2} = \frac{h (b-c)^2}{2} - S_{Y,l} \]  
\[ \frac{\partial I_{XY,r}}{\partial t_w} = \frac{\partial I_{XY,l}}{\partial t_w} = \frac{1}{8} t_w (h-t_2)^2 \]  
(A33)