ABSTRACT
Diffraction of highly-nonlinear transient waves of extreme height by floating bodies is numerically simulated within the scope of potential theory. A finite-element method based on Hamilton’s principle is used to solve the initial-boundary-value problem. The nonlinearities related to the free-surface condition and the fluid domain are fully considered without any approximation. The specified nonlinear waves are obtained numerically from the two-dimensional wave tank simulated by the same finite-element method. A numerical matching scheme is developed to match the long-crested waves at the far field and the three-dimensional diffracted waves in the near field. Numerical results for bottom mounted vertical cylinders are presented.

INTRODUCTION
For the structural design of ships and offshore structures, various environmental factors during their service life are considered. Among them, the wave-induced loads, including diffraction pressure and inertial loads due to motions, are the most important environmental factors to be considered. The existing design practice of floating structures have heavily relied on statistical analysis of the loads under the assumption that the ocean surface can be represented by Gaussian random variables, which can be described by the synthesis of sufficient number of linear wave components with uniformly-distributed random phases. However, there have been many observed occasions where the waves with extreme heights and long crests arise from the sea conditions that such waves should not occur if the wave statistics follows the Gaussian assumptions. With their extreme heights and slopes, they have been called ‘freak waves’, ‘rogue waves’, ‘episodic waves’ or simply ‘wall of water’ (see e.g., Bascom, 1980; Doneal, 1991). This rare event of extreme waves has been blamed for many losses of ships and floating structures in the deep ocean (see e.g., Faulkner, 2001).

Recently, there have been extensive studies to identify the generating mechanism of these extreme waves. There are two different approaches: one is the focusing of linear wave components to build up wave crests at one location; extensive experimental and numerical work has been done along this approach (see, e.g., Zou & Kim, 1995; Chaplin, 1996, Clauss, 1999; Smith & Swan, 2002), and the second approach is to explain the generation of extreme waves by the self-modulation of the wave train due to nonlinear interaction. The type-1 instability of Stokes wave (McClean, 1982), which also is known as Benjamin-Feir instability, plays an important role in triggering the self-modulation. Weakly-nonlinear wave models, such as the cubic nonlinear Schrödinger equation, has been applied to describe the nonlinear interaction involved in the generation and evolution of freak waves (see e.g., Trulsen & Dysthe, 1997; Osborne, 2001; Grue, 2002). Recently, more sophisticated theoretical models have been applied to describe the phenomena more accurately as in Kim & Ertekin (2000).

In addition to the generating mechanism and kinematics of the freak waves, the wave loads due to the interaction between such waves and a floating structure need to consider freak waves in the design of such structures. The main objective of this study is to develop an efficient numerical tool to calculate wave loads due to freak waves impinging on a floating structure with simple geometry.

Recently, Kim et al. (2004) developed a finite-element method to solve the diffraction of steep transient waves by vertical cylinders. The finite-element method is based on Hamilton’s principle and has been successfully applied to some two-dimensional problems (Bai & Kim, 1995; Kim & Bai, 1999; Kim et al., 2003). In three dimensions, the problem has been applied to nonlinear wave-resistance problems (see, Bai et al., 1992, 2003). The same method has been extended to solve the three-dimensional diffraction problems by introducing a new radiation condition. A new numerical scheme was developed to match the Stokes wave solution at the far
field and the diffracted nonlinear wave in the near field. In this new scheme, a buffer subdomain was introduced to obtain a gradual and smooth transition from the near field solution to the Stokes wave at the far field. A similar technique to match the nonlinear near-field solution to the linear far-field solution had been used successfully in the nonlinear wave resistance problem (see, Bai & Kim, 1992; Bai et al, 1992; Bai & Han, 1994).

The same numerical scheme has also been extended to solve the diffraction of transient long-crested waves by floating bodies. The transient wave is generated in a two-dimensional numerical wave flume either by pulsating a pressure patch or the self-modulation of a Stokes wave train. Emphasis has been made to the diffraction of steep waves with wavelengths much longer than the diameter of the vertical cylinder. Although the linear theory predicts negligible wave diffraction, experiments and field observations on offshore structures have shown significant design issues such as wave run up and ringing phenomena at such a set up (e.g., Zou & Kim, 1995; Grue & Husbey, 2002; Chaplin et al., 1997). In this paper, the issue of breaking wave loads is excluded and we consider the diffraction of non-breaking extreme waves due to a floating body of simple geometry. Specifically, the diffraction of steep Stokes waves and focused waves by vertically-wall-sided floating bodies and pontoons are presented after an introduction of the theoretical and numerical aspects of the finite-element method used.

**MATHEMATICAL FORMULATION**

We consider the free-surface flow of an inviscid and incompressible fluid, in water of uniform depth, \( h \). The coordinate system is chosen such that the \( z \)-axis directs against the gravity and the \( Oxy \)-plane is the still-water level unless otherwise stated. The location of the free surface is denoted by \( z = \zeta(x, y, t) \) and the bottom as \( z = -h \). The fluid domain is denoted by \( D \), and the boundaries of the fluid domain, the free surface, sea bottom and vertical cylinder, are denoted by \( S_F \), \( S_B \) and \( S_0 \), respectively. The lateral boundary of the fluid domain should be at infinity. However, for computational purposes, the far-field domain must be truncated at the numerical radiation boundary, \( S_R \), which will be specified later when we discuss some numerical examples. Fig. 1 shows a typical subdivision of the computational domain that is used in the present study.

Hereafter, we nondimensionalize all physical variables such that the mass density of the fluid, \( \rho \), gravitational acceleration, \( g \), and the characteristic length can be taken as unity. The characteristic length is the water depth, \( h \), or the wavelength, \( \lambda_c \), depending on the particular problem, and this will be indicated accordingly.

We assume that the fluid motion is irrotational; therefore the velocity potential, \( \phi(x, y, z, t) \), can be defined to describe the fluid motion. The velocity potential is governed by the Laplace equation in the fluid domain and is subject to appropriate boundary conditions on the boundaries. The governing equation in the fluid domain is therefore given by

\[
V^2 \phi = 0 \quad \text{in } D.
\]  

(1)

The normal velocity of the fluid vanishes on the sea floor:

\[
\frac{\partial \phi}{\partial z} = 0 \quad \text{on } z = -h,
\]  

(2)

and on the cylinder wall, \( S_0 \), i.e.,

\[
\frac{\partial \phi}{\partial n} = 0 \quad \text{on } S_0,
\]  

(3)

where \( n \) is the outward vector normal to the cylinder surface.

On the free surface, \( S_F \), the kinematic and dynamic conditions can be written as

\[
\frac{\partial \zeta}{\partial t} = \frac{\partial \phi}{\partial z} - \frac{\partial \phi}{\partial x} \frac{\partial \zeta}{\partial x} - \frac{\partial \phi}{\partial y} \frac{\partial \zeta}{\partial y} \quad \text{on } S_F.
\]  

(4)
where \( z = \zeta(x, y, t) \) is the free surface and \( R_0 \) is the Bernoulli constant, which can be taken as zero if the \( Oxy \) plane is defined as the still-water level.

The above problem can be completed by specifying an initial condition and a radiation condition at infinity on the horizontal plane. We assume that the free-surface flow is caused by a known incoming wave solution pair \( \zeta_w \) and \( \phi_w \) that satisfies the initial-boundary-value problem described by Eqs. (1), (2), (4) and (5) without in the absence of a structure. Then, at infinity, the following radiation condition should be satisfied:

\[
\phi = \phi_w + O\left( \frac{1}{\sqrt{r}} \right) \quad \zeta \equiv \zeta_w + O\left( \frac{1}{\sqrt{r}} \right) \quad \text{as} \quad r \to \infty,
\]

where \( r = \sqrt{x^2 + y^2} \). It should be noted that a different radiation condition should be used if the fluid domain is partially confined, such as in a wave tank. Since we cannot solve the problem numerically in an infinite fluid domain, the radiation condition at infinity is replaced by numerical matching conditions at the finite truncated boundary, \( S_{fr} \), i.e.,

\[
\phi = \phi_w, \quad \zeta = \zeta_w \quad \text{on} \quad S_{fr},
\]

which is valid only when the minimum radial distance, \( r \), from the cylinder to the radiation boundary, \( S_{fr} \), is sufficiently large because the far-field propagating wave decays as \( O\left( \frac{1}{\sqrt{r}} \right) \). To accelerate the decay and minimize the required distance of the radiation boundary, artificial damping terms are added to the free-surface conditions, Eqs. (4) and (5), in a transition buffer domain, \( S_{fr} \):

\[
\frac{\partial \zeta}{\partial t} = \frac{\partial \phi}{\partial x} - \frac{\partial \phi}{\partial x} \zeta - \frac{\partial \phi}{\partial y} \zeta - \mu(x, y) \left( \zeta - \zeta_w \right) \quad \text{on} \quad S_{fr},
\]

\[
\frac{\partial \phi}{\partial t} = -\frac{1}{2} \nabla \phi \cdot \nabla \phi + g(\zeta - R_0) - \mu(x, y) \left( \phi - \phi_w \right) \quad \text{on} \quad S_{fr},
\]

where \( \phi = \phi(x, y, \zeta, t) \) and \( \phi_w = \phi_w(x, y, \zeta_w, t) \). The damping function \( \mu(x, y) \) varies from zero at the inner boundary of \( S_{fr} \) to a constant value at the outer area of \( S_{fr} \). A more specific shape of the damping function will be provided later in discussing the numerical examples.

**FINITE ELEMENT METHOD**

Following Bai & Kim (1995), the initial-boundary-value problem, given by Eqs. (1) through (9), is solved by a finite-element method based on Hamilton’s principle.

\[
L = \dot{\phi} \zeta + \frac{1}{2} \int_{-h}^h \nabla \phi \cdot \nabla \phi \, dz - \frac{1}{2} \zeta^2 - \ddot{\phi} \zeta + \ddot{\phi}.
\]

The additional damping terms in the free-surface conditions are treated as a pressure and mass flux distribution on the free surface, which are written as

\[
\ddot{p} = \mu(x, y) \left( \phi - \phi_w \right),
\]

\[
\ddot{q} = \mu(x, y) \left( \zeta - \zeta_w \right),
\]

respectively. It can be shown that the stationary condition of the time integration of the Lagrangian with respect to the variations of \( \phi, \dot{\phi}, \zeta, \zeta \) lead to the governing equation (1) and boundary conditions (2), (3), (8) and (9).

We restrict the geometry of the floating structure to consist of vertically-wall-sided cylinders. But we allow the discontinuity of geometry along the vertical direction, as in the case of a pontoont type structure shown in Figure 2(a). Since the fluid domain varies because of the surface elevation, \( \zeta(x, y, t) \), it is more convenient to adopt a transformed coordinate system \((x, y, \gamma, t)\) for each layer \( L^\alpha \), defined in Fig. 2(b), such that

\[
\phi(x, y, z, t) = \phi^\alpha(x, y, \gamma, t), \quad z \in L^\alpha, \quad \alpha = 1, 2, 3,
\]

where

\[
\gamma_1 = \frac{z + h}{h - h_1}, \quad -h \leq z < -h_1,
\]

\[
\gamma_2 = \frac{z + h_1}{h_1 - h_2}, \quad -h_1 \leq z < -h_2,
\]

\[
\gamma_3 = \frac{z + h_2}{h_2 - \zeta}, \quad -h_2 \leq z \leq \zeta.
\]

In the transformed domain, the height of the domain is unity. We can then separate the depth-wise and horizontal variations of the velocity potential as

\[
\phi^\alpha(x, y, \gamma, t) = \sum_{m=1}^{M_\alpha} f_m^\alpha(\gamma) N_m(x, y, t),
\]

\[
\phi_w^\alpha(x, y, t) = \sum_{i=1}^{N_\alpha} \phi_m^\alpha(\gamma) N_i(x, y),
\]

where \( \{f_1^\alpha(\gamma), f_2^\alpha(\gamma), f_3^\alpha(\gamma), ..., f_{M_\alpha}^\alpha(\gamma)\} \) is the set of interpolation functions in the vertical direction, and the set of the horizontal interpolation functions, \( \{N_1(x, y), N_2(x, y), ..., N_{N_\alpha}(x, y)\} \), is the two-dimensional finite-element shape functions defined in \( D \), and \( M_\alpha \) and \( N_\alpha \) denote the number of interpolation functions in the vertical direction and on the horizontal plane, respectively. The free-surface elevation is also expanded by the same interpolation function:

\[
\zeta(x, y, t) = \sum_{i=1}^{N_\alpha} \zeta_i(\gamma) N_i(x, y)
\]
The interpolation functions for each layer are given by

\[ f_1^\alpha = \gamma; \quad f_2^\alpha = 1 - \gamma; \]
\[ f_m^\alpha = \sin((m-3)\pi\gamma), \quad m > 3. \] (17)

For the horizontal interpolation function, we use a linear interpolation function for a three-node triangular element. A typical finite-element mesh around the vertical cylinder is shown in Fig. 3. Once the spatial discretization has been made by the finite-element method, the Laplace equation, Eq. (1), reduces to a system of algebraic equations for the velocity potential at the internal nodes. An iterative scheme (Jacobi conjugate gradient method) is used to solve the system of equations. The matrix involved in the algebraic equations is highly sparse. Only the non-zero entries of the sparse matrix is stored. The free-surface conditions given by Eqs. (4) and (5) lead to time-evolution equations for the surface elevation and the velocity potential on the free surface. They are first-order differential equations and are integrated by the 4th order Runge-Kutta method.

For the damping function \( \mu(x, y) \) that appears in Eqs. (8) and (9), the following expression is used:

\[ \mu(x, y) = \mu_0 \left[ \mu_x(x) + \mu_y(y) - \mu_z(x) \mu_y(y) \right], \] (18)

where

\[
\begin{align*}
\mu_x(x) &= \begin{cases} 
\cos^2 \left( \frac{\pi(x-x_1)}{2(x_2-x_1)} \right), & x_1 < x \leq x_2; \\
0, & x_2 < x \leq x_3; \\
\cos^2 \left( \frac{\pi(x-x_3)}{2(x_4-x_3)} \right), & x_3 < x \leq x_4.
\end{cases} \\
\mu_y(y) &= \begin{cases} 
\cos^2 \left( \frac{\pi(y-y_1)}{2(y_2-y_1)} \right), & y_1 < y \leq y_2; \\
0, & y_2 < y \leq y_3; \\
\cos^2 \left( \frac{\pi(y-y_3)}{2(y_4-y_3)} \right), & y_3 < y \leq y_4.
\end{cases}
\end{align*}
\] (19) (20)

The far-field wave solution \( \zeta_w \) can represent any nonlinear transient wave solution. Three different incoming wave solutions have been considered here: the fully-nonlinear Stokes wave, a self-modulated freak wave, and a freak wave generated by phase focusing.

For a floating structure with multiple surface piercing columns, waves can be trapped between the columns and cause impact loads on the top structures. Since the trapping modes take several wave periods to build up, regular wave trains with moderate heights can be more damaging compared to freak waves with a single or a few number of wave crests. In this regard, the fully-nonlinear Stokes wave can be a good mathematical model for a regular wave train in a wave tank test, or a swell in the case of a fully-developed sea.

**DIFFRACTION BY STOKES WAVES**

For a floating structure with multiple surface piercing columns, waves can be trapped between the columns and cause impact loads on the top structures. Since the trapping modes take several wave periods to build up, regular wave trains with moderate heights can be more damaging compared to freak waves with a single or a few number of wave crests. In this regard, the fully-nonlinear Stokes wave can be a good mathematical model for a regular wave train in a wave tank test, or a swell in the case of a fully-developed sea.
The Stokes wave solution has been calculated here numerically by solving the problem by the two-dimensional version of the present finite-element method.

A Stokes wave can be defined by the wave height, $H$, water depth, $h$, and wavelength, $\lambda$. Following Cokelet (1977), the length scale is nondimensionalized such that the wavelength is $2\pi$ in the nondimensionalized coordinate. The water depth usually used for the definition of the Stokes wave is different from the still-water depth $h$ and is defined by

$$d = \frac{Q}{C},$$

where $Q$ is the mass flux and $C$ the celerity of the Stokes wave. The celerity, $C$, of the Stokes wave is defined in the reference frame where there is no circulation, or the increase in the velocity potential over the length of the wave:

$$\int_0^{2\pi} u(x,z) \, dx = \phi(2\pi,z) - \phi(0,z) = 0. \tag{22}$$

On the other hand, when we are interested in Stokes waves generated by a wave maker, the reference frame should be the one where there is no net mass flux at any vertical surface along the wave train, i.e.,

$$\int_0^{2\pi} \int_{-h}^{\zeta} u(x,z) \, dz \, dx = 0. \tag{23}$$

In the reference frame defined by the relation (22), there is non-zero mass flux toward the direction of wave propagation. We denote the mean velocity due to the mass flux by $\overline{u}$, which is defined by

$$\overline{u} = \frac{1}{2\pi d} \int_0^{2\pi} \int_{-\zeta}^{\zeta} u(x,z) \, dz \, dx. \tag{24}$$

As a result, the Stokes wave solution at the ‘wave-maker’ reference frame can be obtained by adding uniform backward flow with flow velocity, $-\overline{u}$, to the Stokes wave solution in the reference frame defined by (22). Then the celerity of the Stokes wave solution in the new reference frame, $C'$, and the water depth, $h$, have the following relation with the celerity, $C$, and water depth, $d$, in the old reference frame:

$$C' = C - \overline{u},$$

$$h = d + R_0 + \frac{\overline{u}^2}{2}, \tag{25}$$

where $R_0$ is the Bernoulli constant shown in Eq. (5), in the reference frame where $z = -d$ denote the sea bottom.

The same finite-element method and Newton’s method that were used for the solitary wave problem by Bai & Kim (1995) have been used here for Stokes waves. We first obtain the Stokes wave solution in the original Cokelet’s reference frame and then transform it to the wave-maker reference frame. A nonlinearity parameter, $\varepsilon$, is introduced by following Cokelet (1977):

$$\varepsilon = \frac{q_{\text{crest}}^2 - q_{\text{trough}}^2}{C^4}, \tag{26}$$

which varies from 0 for a linear wave to 1.0 at the breaking limit. In Eq. (26), $q_{\text{crest}}$ and $q_{\text{trough}}$ denote the fluid particle velocities at the crest and trough of the Stokes wave, respectively.

The Stokes wave solution is obtained for given values of $\varepsilon$ and $kd$. The relation between the wave amplitude $H/2 = A$ and the celerity, $C$, are compared with the semi-analytic solution of Cokelet (1977) in Fig. 3. The agreement is very good. Faster convergence is attained in shallower depth, or for higher values of $e^{-kd}$.

In Fig. 5, the same relation is plotted along the constant value of two water depths $d$ and $h$, and two wave periods $T$ and $T_1$, which are defined by

$$T = \frac{2\pi}{kC}, \quad T_1 = \frac{2\pi}{kC'}, \tag{27}$$

It can be seen that the difference between the two reference frames is significant for steep waves in the intermediate water depth. At the deep water, where $e^{-kd} = 0$ and there is no mass flux, and at the solitary wave limit, $e^{-kd} = 1$, where the wave length is infinite, the Stokes wave solutions at the two reference frames become identical.
A Numerical Study of Nonlinear Diffraction Loads on Floating Bodies due to Extreme Transient Waves

The Stokes wave input has been used to study the wave trapping between a three-leg GBS operating in the North Sea by Kim et al (2004). The numerical solution agrees very well with the wave tank test by Swan et al. (1997). Very steep trapped wave crest, which is more than three times higher than the crest height of the incoming Stokes wave, has been observed both in the numerical simulation and model test. Fig. 6(a) shows the snapshot of the surface profile at the moment of maximum wave elevation. Clearly seen is the short diffracted waves with short wavelength interacting with the incoming Stokes wave, which cannot be explained by the linear theory. The time history of the surface elevation measured at the location of maximum elevation is plotted in Fig. 6(b) compared with the measured data. Excellent agreement can be seen.

**FREAK WAVE BY SELF-MODULATED STOKES WAVE**

The self-focused waves of extreme height can be generated from a perturbed Stokes wave train. Fig. 8 shows evolution of a Stokes wave train that was slightly modulated by superimposing a side-band disturbance at \( t = 0 \). Stokes wave of \( kA = 0.1 \) is modulated by adding sinusoidal waves with wave numbers 90% and 110% of the carrier wave, and with amplitudes 5% of the carrier wave. The mechanical energy transfer from the carrier Stokes wave to the side-band wave components results in a strongly modulated wave packet with extreme wave height. Fig. 8(c) shows the strongly modulated wave after evolution of 320 periods. The difference between wave elevation at wave trough and crest is about 2.5 times of the initial wave height. The wave profile also shows a severe asymmetry in the crest and trough height. When there is a freak wave, a ‘deep hole’ in the ocean surface is often observed just before the extreme crest of the freak wave erupts (see, e.g., Bascom, 1980 and Faulkner, 2001). Similar phenomenon has been observed in the numerical simulation of self-modulation. Fig. 8(b) shows the wave profile at \( t = 319 T \), or one period ahead of the moment when the wave elevation reaches the maximum value and shown in Fig. 8(c). As opposed to usual nonlinear waves with flat trough, the focused wave shows a deep trough that can be interpreted as ‘deep-hole’ at this time step. One of the possible explanation of this phenomena can be the ‘un-locking’ of the second harmonic component of the Stokes wave from the primary carrier wave. The second harmonic component of the Stokes wave travels in the same speed of the primary wave, or locked to the primary wave, and always makes the crest sharper and trough flatter in the regular wave train. But in the highly-modulated freak wave, the second harmonic can be unlocked and travel with its slower free wave speed.

To show the time evolution of amplitude and phase of each wave component and possible unlocking of the second harmonic component, the amplitude of the spatial Fourier components \( a_n(t) \) and its celerity \( c_n(t) \) of the modulated Stokes wave shown in Figure 8 are investigated. They are defined by
\[ a_n(t) = \frac{1}{2\pi} \int_{-\pi}^{\pi} \zeta(x,t) \exp(-inx) \, dx, \quad (28) \]

\[ c_n(t) = \frac{1}{2T} \Im \left( \int_{-T}^{T} \frac{1}{na_n(t')} \frac{d}{dt'} a_n(t') \, dt' \right). \quad (29) \]

Figure 7  Time history of wave elevation at the location of maximum elevation shown in Figure 6. Solid line: wave elevation without cylinders; Dotted line: computed wave elevation with the cylinders; Symbols: measured wave elevation (Swan et al, 1997)

The celerity is averaged over two periods of the carrier Stokes wave to remove the fluctuations that have been observed otherwise. The amplitudes of the Fourier components and celerities of the same wave system given in Figure 8 are shown in Fig. 9(a) and 9(b), respectively. Also shown is the spatial maximum and minimum of the wave elevation in Fig. 9(c). The energy of the carrier wave \( (n = 10) \) is transferred to the side-band instability \( (n = 9, 11) \) seeded initially. The energy is also transferred to the other side-band components, \( n = 8 \) and 12. Actually, the wave components \( n = 8 \) and 12 are closer to the most unstable mode of the Type-1 instability described in McClean (1982). After \( t = 320T \), permanent down-shifting of the energy to the lower side-band instabilities \( (n = 8 \) and 9) are observed.

When the energy transfer from the primary mode, \( n = 10 \), to the other modes are maximal and when the strong modulation of the Stokes wave leads to the peak values of the maximum and minimum surface elevation, the celerity of the second harmonic components are dropped to certain values which are close to the celerity of the free waves of the same length. In other words, the phase of the second harmonic components are unlocked from the primary components.

Since the secondary wave components are unlocked from the primary wave components during the emerging of the freak waves, the kinematics of each wave components will be similar to that of the freak waves generated by phase focusing, which will be discussed in the next section.

DIFFRACTION OF FREQUENCY-FOCUSED FREAK WAVE

Waves are generated by a pulsating pressure patch in a half-infinite domain where a vertical wall is placed at \( x = 0 \). The pulsating pressure is given by

\[ \hat{p}(x,t) = \sum_{n=1}^{N} A_n H(\pi/k_n - x) \cos^2(2k_n x \sin(\omega_n t - \delta_n)) \quad (30) \]

where \( H(x) \) is the Heaviside function that equals to zero when \( x < 0 \) and unity when \( x > 0 \). Further, \( k_n \) is the wave number of a linear Stokes wave for a given wave frequency \( \omega_n \). When the magnitude of pressure is small, the linear solution of the waves generated by the pulsating pressure patch can be derived from an analytic solution given in Wehausen & Laitone (1960):

\[ \zeta_L(x,t) = \sum_{n=1}^{N} P_n A_n \sin(k_n x - \omega_n t - \delta_n) \quad (31) \]

where

\[ P_n = \frac{2}{k_n} \frac{\sinh(2k_n)}{\omega_n^2 + \sinh^2(k_n)} \quad (32) \]
The phase of each component waves, \( \delta_n \), are selected to make crests of all wave components arrive at a given location, \( x = x_C \) at a given time \( t = t_C \), i.e., \( \delta_n = k_n x_C - \omega_n t_C \). When the generated wave is not small, its wavelength and phase speed differ from the linear solution. An iterative scheme described in Chaplin (1996) has been used to consider the phase change due to nonlinearity. Fig. 10 shows the time history and instantaneous profile of wave elevation compared with the experimental data. This wave corresponds to the 'case 765' in Chaplin et al. (1997) that was used to study the ringing of a vertical cylinder. The agreement between the experimental data and numerical results is good. The wave profile near the sharp crest looks similar to the freak wave by self-focusing shown in Fig. 8(c).
The diffraction of the focused wave by a circular cylinder has been simulated by Kim et al. (2003) and the wave-exciting moment is compared with the experimental result of Chaplin et al. (1997). Fig. 11 shows the time history of the wave-exciting moment. The numerical result agrees well until it reaches the maximum value at around \( t/\sqrt{h/g} = 3.5 \). After then, the numerical value slightly underestimates the magnitude of the wave load. Presumably, this is due to the viscous effect in the experiments conducted with small model size. At around \( t/\sqrt{h/g} = 4.5 \) short-duration increase of wave load can be observed both in the numerical and experimental result. This is known as the ‘secondary oscillation’ and plays an important role in the ringing response of the structure (Chaplin et al., 1997; Grue & Huseby, 2002).

A vertical cylinder of a simple geometry that has beam-length ratio similar to FPSO floating structure is considered next (see Fig. 12). The wave exciting force due to the freak wave profile given in Fig. 10 is calculated for various heading angles.

As shown in Fig. 13, the maximum force occurs at the heading angle of 100 degrees. Significant secondary oscillation at around \( t/\sqrt{h/g} = 4.5 \) can be seen. Fig. 14 shows the surface elevation around the cylinder at 120° heading. Higher run-up at the stern area is observed than the bow area. Also observed is the nearly vertical wave slope around the stern area at \( t/\sqrt{h/g} = 4.1 \).

**CONCLUSIONS**

A finite-element method has been proposed to simulate nonlinear wave-body interaction between steep nonlinear waves with extreme height and floating structures. Numerical results for the vertical cylinder have been presented. Nonlinear wave run up and loadings have been calculated and compared to the available experimental results with good agreements. Extension of the present numerical method to simulate nonlinear wave diffraction around more practical floating structures is in progress.

**REFERENCES**


