Buckling and Ultimate Strength Assessment of FPSO Structures

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ABSTRACT

Floating production, storage and offloading systems (FPSOs) have been widely used for the development of offshore oil and gas fields because of their attractive features. They are mostly ship-shaped, either converted from existing tankers or purposely built, and the hull structural scantling design for tankers may be applicable to FPSOs. However, FPSOs have their unique characteristics. FPSOs are sited at specific locations with a dynamic loading that is quite different from those arising from unrestricted service conditions. The structures are to be assessed to satisfy the requirements of all in-service and pre-service loading conditions. The fundamental aspects in the structural assessment of FPSOs are the buckling and ultimate strength behaviors of the plate panels, stiffened panels and hull girders. The focus of this paper is to address the buckling and ultimate strength criteria for FPSO structures. Various aspects of the criteria have been widely investigated, and the results of the design formulae proposed in this paper have been compared to a very extensive test database and numerical results from nonlinear finite element analysis and other available methods. The procedures presented in this paper are based on the outcomes of a series of classification society projects in the development of buckling and ultimate strength criteria and referred to the corresponding classification society publications.

KEY WORDS: Floating production, storage and offloading systems (FPSOs); Buckling and Ultimate Strength; Progressive Collapse Analysis; Structural Design Criteria; Nonlinear Finite Element Analysis

NOMENCLATURE

\( A \) = Total sectional area of longitudinal
\( A_e \) = Effective sectional area of longitudinal
\( A_s \) = Sectional area of longitudinal, excluding the associated plating
\( b_f, t_f \) = Flange width and thickness of stiffener, see Figure 2
\( b_1 \) = Smaller outstand of flange of stiffener, see Figure 2
\( B \) = Unsupported spacing of the supporting girders
\( C_1 \) = Stiffener’s type-dependent coefficient for elastic buckling stress of the plating in edge shear
\( C_2 \) = Stiffener’s type-dependent coefficient for elastic buckling stress of the plating in compression
\( C_E \) = Coefficient as defined in Section 3.4
Moment adjustment coefficient, which may be taken as 0.75
Coefficient as defined in Section 2.3.1
Coefficient as defined in Section 2.3.1
Coefficient as defined in Section 2.3.1
Moment adjustment coefficient, corresponding to Figure 2
Modulus of elasticity
Moment of inertia of longitudinal or stiffener, accounting for the effective width, \( s_e \)
Moment of inertia of the transverse main supporting members, including effective width of plating
Moment of inertia of the stiffeners, including the effective width of plating
Polar moment of inertia of the stiffener, excluding the associated plating
Moment of inertia of the longitudinal or stiffener, excluding the associated plating
St. Venant torsion constant for the stiffener cross-section, excluding the associated plating
Boundary condition dependent constant
Length of long plate edge or unsupported span of the longitudinal or stiffener, see Figure 21
Bending moment on longitudinal induced by lateral pressure, equal to \( q_E t^2 / 12 \)
Still water bending moment and wave induced bending moment of hull girder
Hull girder ultimate strength
Number of half-waves of associated plating that yield the smallest \( \sigma_E \)
Exponential value on \( E \) to denote the post-buckling behavior, which may be taken as 2.0 for steel
Total compressive load on stiffener using full width of associated plating
Proportional linear elastic limit of the structure, which may be taken as 0.6 for steel
Lateral pressure for the region considered
Ultimate lateral pressure
Radius of gyration of area, \( A_e \)
Length of short plate edge or longitudinal spacing
Effective width of plating
Effective width of plating corresponding to \( E \)
Effective breadth of plating
Effective section modulus of the longitudinal at flange, accounting for the effective breadth, \( s_{ey} \), see Figure 21
Thickness of plating
Initial deflection of plating
Initial deflection of longitudinal
Centroid of stiffener, see Figure 2
L/s, plating aspect ratio
Plating slenderness ratio
Plating slenderness ratio corresponding to \( E \)
Ratio of edge stresses, equal to \( \sigma_{min} / \sigma_{max} \)
Nominal calculated compressive stress, equal to \( P/A \)
In-plane bending stress in longitudinal direction (\( i=x \)) and transverse direction (\( i=y \))
Bending stress applied to longitudinal, equal to \( M/S_M \)
In-plane bending stress in longitudinal direction and transverse direction
Stress for beam-column buckling of longitudinal corresponding to \( E \)
Stress for torsional-flexural buckling of longitudinal corresponding to \( E \)
Stress for local buckling of longitudinal corresponding to \( E \)
Critical buckling stress of plating for uniaxial compression in the longitudinal direction
Critical buckling stress of plating for uniaxial compression in the transverse direction
Critical buckling stress of longitudinal in axial compression
Critical buckling stress for associated plating corresponding to n-half waves
Critical torsional-flexural buckling stress with respect to axial compression of longitudinal, including its associated plating
Equivalent stress according to von Mises, equal to \( (\sigma_i^2 - \sigma_{max} \sigma_{y, max} + \sigma_{max}^2 + 3 \sigma_i^2)^{\frac{1}{2}} \)
Euler’s buckling stress of longitudinal
Elastic local buckling stress, which is the smaller value of Euler’s buckling stresses of web, \( \sigma_{EL} \), and flange plate, \( \sigma_{EF} \)
Elastic buckling stress of plating in longitudinal direction (\( i=x \)) and transverse direction (\( i=y \))
Elastic torsional-flexural buckling stress with respect to the axial compression of longitudinal, including its associated plating
Specified minimum yield point of material
Ultimate strength with respect to uniaxial stress in the longitudinal direction
Ultimate strength with respect to uniaxial stress in the transverse direction

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\( \sigma_{CP} \) = Critical buckling strength of an unstiffened plate corresponding to \( \bar{E} \)

\( \sigma_{UP} \) = Ultimate strength of an unstiffened plate corresponding to \( \bar{E} \)

\( \sigma_{CA} \) = Critical buckling stress of longitudinal corresponding to \( \bar{E} \)

\( \sigma_{CL} \) = Critical local buckling stress of a stiffener corresponding to relative strain ratio, \( \bar{E} \)

\( \sigma_{r,\text{max}} \) = Maximum compressive stress in the longitudinal direction

\( \sigma_{y,\text{max}} \) = Maximum compressive stress in the transverse direction

\( \tau \) = Edge shear stress

\( \tau_C \) = Critical buckling stress for edge shear

\( \tau_U \) = Ultimate strength with respect to edge shear

\( \tau_o \) = Shear strength of material, equal to \( \sigma_o / \sqrt{3} \)

\( \varphi \) = Coefficient to reflect interaction between longitudinal and transverse stresses (negative values are acceptable)

\( \phi_o \) = Load combination factor

\( \nu \) = Poisson’s ratio, 0.3 for steel

\( \gamma_o, \gamma_s, \gamma_u \) = Partial safety factors corresponding to still water bending moment, wave-induced bending moment and hull girder ultimate strength

\( \eta \) = Maximum allowable strength utilization factor

\( \varepsilon \) = Axial strain

\( \varepsilon_o \) = Initial yield strain

\( \bar{\varepsilon} \) = Relative strain ratio, equal to \( \varepsilon / \varepsilon_o \)

\( \Gamma \) = Warping constant

1 INTRODUCTION

Floating production, storage and offloading systems (FPSOs) have been used widely for the development of offshore oil and gas fields because of their attractive features such as large work area and storage capacity, mobility (if desired), relatively low cost construction and good stability. They are mostly ship shaped, either converted from existing tankers or purposely built, and the hull structural scantling design for tankers may be applicable to FPSOs. However, FPSOs have their unique characteristics. FPSOs are sited at specific locations with a dynamic loading that is quite different from those arising from unrestricted service conditions. The structures are to be assessed to satisfy the requirements for all in-service and pre-service loading conditions. The in-service conditions include:

- In-place intact condition
  - Design operating condition
  - Design environmental condition

- Loadout condition
- Transit condition
- Installation condition

For each load condition, the allowable strength utilization factor (factor of safety) should be defined as specified in the ABS Rules for Building and Classing Mobile Offshore Units (2001) and the Guide for Building and Classing Floating Production Installations (2003).

Strength of FPSOs is one of the most important controls during design (the other one can be fatigue control). Similar to ships, the strength of FPSOs includes three aspects, which are global longitudinal strength, transverse strength and local strength, as described in Simonsen et al. (2003). Practically they can be assessed by either prescriptive rule or finite element analysis (FEA), within the theory of elasticity in general. Ultimate strength of both structural components (plate and stiffened panel) and structural systems are then steps forward considering material and geometric behavior beyond the elastic region, namely, material and geometric nonlinearity.

With the development of computers and robust methods for nonlinear FEA, there has been a tremendous increase in the number of studies of structures under plastic or elasto-plastic behavior. Even with today’s computers and software, non-linear FEA of ship structures is still complex. Considerable effort is therefore devoted to the development of simplified methods for rapid structural assessment and design analysis, instead of lengthy and complex nonlinear FEA, as mentioned in Kaminski et al. (2000).

During the past years, ABS has been working on a series of projects to develop buckling and ultimate strength criteria to be applied to the design of offshore structures. The method presented in this paper is based on the outcome of those ABS projects.

It has been generally accepted that the longitudinal strength of ship structures is well represented by the strength of representative longitudinally stiffened panels, which are composed of plate panels, longitudinal stiffeners and transverse frames, as illustrated in Figure 1. In Section 2 of this paper, buckling and ultimate strength criteria of plate and stiffener panel are first introduced. Then, the analysis procedure of hull girder ultimate strength, as a system is composed of plates and stiffener panels, is presented in Section 3, including load-end shortening curve for plate and stiffened panel. In Section 4, hull girder ultimate strength of two ships is calculated and compared with different methods as a validation of the method.
presented in this paper. Finally, the conclusion of this paper is summarized in Section 5.

2 BUCKLING STRENGTH OF PLATE AND STIFFENED PANELS

2.1 General

For plates and stiffened panels in compression, the basic load case usually consists of the following loads applied simultaneously:

- Longitudinal compression arising from the overall hull girder bending
- Transverse compression arising from the in-plane pressure loading
- Local bending arising from the lateral pressure

Large research efforts on the buckling and collapse behavior of plate or stiffened panels have been made since the 1970’s. Some earlier researches on these topics are, Smith (1975), Calsen (1980), Smith et al. (1987, 1991), etc. Kaminski et al. (2000) made a good review of the recent activities in this area. As a consequence, many authorities and classification societies also published their requirements and analysis procedures for buckling strength of plates and stiffened panels.

Ship hull structures are constructed mainly of plates and stiffened panels, as seen in Figure 1. Stiffeners in the stiffened panels are usually installed equally spaced, parallel or perpendicular to panel edges in the direction of dominant load and are supported by heavier and more widely-spaced ‘deep supporting members’, i.e., girders. The section dimensions of a stiffener are defined in Figure 2. The stiffeners may have strength properties different from those of the plate.

Figure 1: Typical stiffened panel

Figure 2: Sectional dimensions of a stiffened panel

The plate and stiffened panel criteria account for the following load and load effects. The symbols for each of these loads are shown in Figure 3.

- Uniform in-plane compression, $\sigma_{ax}$, $\sigma_{ay}$ (If uniform stress, $\sigma_{ax}$ or $\sigma_{ay}$, is tensile rather than compressive it may be set equal to zero)
- In-plane bending, $\sigma_{bx}$, $\sigma_{by}$
- Edge shear, $\tau$
- Lateral loads, $q$
- Combinations of the above

Figure 3: Loads and load effects on a stiffened panel
2.2 Plate

2.2.1 Buckling State limit

A large amount of effort has been carried out over the past three decades and different interaction equations have been suggested. The buckling state limit for plate panels between stiffeners in this paper is defined by the following equation (ABS, 2004a):

\[
\left( \frac{\sigma_{x_{\text{max}}}}{\eta C_{r}} \right)^{2} + \left( \frac{\sigma_{y_{\text{max}}}}{\eta C_{y}} \right)^{2} + \left( \frac{\tau}{\eta C_{\tau}} \right)^{2} \leq 1 \tag{1}
\]

where \( \eta \) is the maximum allowable strength utilization factor. The critical buckling stresses are specified below.

2.2.1.1 Critical buckling stress for edge shear

The critical buckling stress for edge shear, \( \tau_{C} \), may be taken as:

\[
\tau_{C} = \begin{cases} 
\tau_{E} & \text{for } \tau_{E} \leq \tau_{o} \tau_{o} \left[ 1 - P_{y} \left( 1 - P_{x} \right) \right] & \text{for } \tau_{E} > \tau_{o} \tau_{o} 
\end{cases} \tag{2}
\]

where,

\[
\tau_{E} = k_{s} \frac{\pi^{2} E}{12(1 - v^{2})} \left( \frac{t}{s} \right)^{2}
\]

\[
k_{s} = \begin{cases} 
4.0 \left( \frac{s}{l} \right)^{2} + 5.34 & \text{for } l = \text{angles or tee stiffeners} \\
1.0 & \text{for } l = \text{flat bars or bulb plates}
\end{cases}
\]

\( C_{l} = 1.1 \) for plate panels between angles or tee stiffeners
\( = 1.0 \) for plate panels between flat bars or bulb plates and web plate of stiffeners

2.2.1.2 Critical buckling stress for uniaxial compression and in-plane bending

The critical buckling stress, \( \sigma_{C_{i}} (i = x \text{ or } y) \), for plates subjected to combined uniaxial compression and in-plane bending may be taken as:

\[
\sigma_{C_{i}} = \begin{cases} 
\sigma_{E_{i}} & \text{for } \sigma_{E_{i}} \leq P_{x} \sigma_{o} \\
\sigma_{o} \left[ 1 - P_{y} \left( 1 - P_{x} \right) \frac{\sigma_{o}}{\sigma_{E_{i}}} \right] & \text{for } \sigma_{E_{i}} > P_{x} \sigma_{o}
\end{cases} \tag{3}
\]

where,

\[
\sigma_{E_{i}} = k_{s} \frac{\pi^{2} E}{12(1 - v^{2})} \left( \frac{t}{s} \right)^{2}
\]

For loading applied along the short edge of the plating (long plate):

\[
k_{s} = C_{1} \begin{cases} 
8.4 & \text{for } 0 \leq \kappa \leq 1 \\
7.6 - 6.4\kappa + 10\kappa^{2} & \text{for } -1 \leq \kappa < 0
\end{cases}
\]

For loading applied along the long edge of the plating (wide plate):

\[
k_{s} = C_{2} \begin{cases} 
1.0875 \left( 1 + \frac{1}{\alpha^{2}} \right)^{2} & \text{for } \kappa < 1/3 \text{ and } 1 \leq \alpha \leq 2 \\
1.0875 \left( 1 + \frac{1}{\alpha^{2}} \right) - 9 \frac{1}{\alpha} & \text{for } \kappa < 1/3 \text{ and } \alpha > 2 \\
\left( 1 + \frac{1}{\alpha^{2}} \right) (1.675 - 0.675\kappa) & \text{for } \kappa \geq 1/3
\end{cases}
\]

where,

\[
\kappa = \frac{\sigma_{/m}}{\sigma_{/m}} , \text{ ratio of edge stresses}
\]

\( \sigma_{/m} \) and \( \sigma_{/m} \) are in-plane maximum and minimum stresses, as defined in Figure 3

\( C_{2} = 1.2 \) for plate panels between angles or tee stiffeners
\( = 1.1 \) for plate panels between flat bars or bulb plates and web plates

2.2.1.3 Comparison Study – Buckling Strength

Test Database

The plate test database consists of 206 datasets for plates under uniaxial compression along their short edges and 15 datasets for plates under uniaxial compression along their long edges. The data were collected from numerous publications. Unfortunately, no records related to plates under edge shear loading were included in the database.

Comparison Basis

Comparisons between the strength predictions of the existing formulations and test data are generally conducted on the basis of maximum allowable strength utilization factor excluded, i.e., \( \eta \) is set to unity. In all cases, measured values of geometry and material properties were input to the strength equations. For single-acting loading conditions such as pure compression, the resultant strength was taken as the predicted value for the model in question. For assessing the accuracy in terms of ‘Modeling Uncertainty’, the test result is then divided by the value from the strength.
formulations. The mean of all of these modeling uncertainty values is then calculated along with the standard deviation in order that the COV (coefficient of variation) can be determined as the ratio of standard deviation to the mean. The mean and COV provide the statistics by which the accuracy of the formulation can be quantified.

Comparison Results

- Long Plates

Figure 4 shows the relationship between $\sigma_{Cx}$ and slenderness ratio $\beta$ for the long plates according to the formulations of the ABS Buckling Guide (ABS, 2004a), DnV CN30.1 (DnV, 1995), API Bulletin 2V (API, 2000) and compared to the test data. Figure 5 shows the distribution of modeling uncertainty. Table 1 gives the statistical characteristics of the modeling uncertainty.

The mean of the modeling uncertainty in all three codes is greater than one and its COV is approximately 30%. The relatively large COV is not surprising because the test data has been collected from many different published sources. In general, the buckling strength evaluated by the three codes is conservative as compared to the test data.

| Table 1: Mean/COV of Modeling Uncertainty of Long Plates |
|----------------|----------------|----------------|
|                | ABS Buckling  | DnV CN30.1     | API Bulletin |
| Mean           | 1.1149        | 1.1731         | 1.0747       |
| COV            | 0.2845        | 0.2723         | 0.3050       |

- Wide Plates

Figures 6 graphically shows the relationship between $\sigma_{Cy}$ and $\beta$ for the wide plates for two different aspect ratios according to the formulation of the ABS Buckling Guide (ABS, 2004a), DnV CN30.1 (DnV, 1995), API Bulletin 2V (API, 2000) and compared to the test data. The results obtained from the formulae of the three codes are in close agreement with the test data.

Figure 7 shows the comparison of buckling coefficient for wide plates subjected to non-uniform compression on the long edges. The buckling coefficient value obtained from the ABS Buckling Guide (ABS, 2004a) is somewhat different from those based on API Bulletin 2V (API, 2000) and DnV CN30.1 (DnV, 1995) when the stress ratio is negative. This is because the formula of the buckling coefficient for wide plate in the ABS Buckling Guide (ABS, 2004a) is derived by the linear interpolation at three stress ratios, e.g., $\kappa=1, 0.33$ and $-1$, based on the same principle as in ABS Steel Vessel Rules (ABS, 2004b).

- Edge Shear Loading
2.2.2 Ultimate strength under combined in-plane stresses and lateral pressure

2.2.2.1 Criteria

The ultimate strength for a plate between stiffeners subjected to combined in-plane stresses and lateral pressure is required to satisfy the following equation:

$$\left(\frac{\sigma_{x_{\text{max}}}}{\eta \sigma_{C_x}}\right)^2 + \phi \left(\frac{\sigma_{y_{\text{max}}}}{\eta \sigma_{C_y}}\right)^2 + \left(\frac{\tau}{\eta_{C_{\tau}}}\right)^2 \leq 1 \quad (4)$$

where,

$$\phi = 1.0 - \beta / 2$$

$$\sigma_{y_{\text{max}}} = C_y \sigma_o \geq \sigma_{C_y}$$

$$C_y = \begin{cases} 
2/\beta - 1/\beta^2 & \text{for } \beta > 1 \\
1.0 & \text{for } \beta \leq 1
\end{cases}$$

When the plate is under combined in-plane stress and lateral pressure, Yao et al (1996, 1997, 1999) published a series of papers to clarify the behaviors of buckling/plastic collapse and strength of ship bottom plates subjected to combined bi-axial thrust and lateral pressure. They considered a portion of continuously stiffened plates for their analysis taking into account symmetry conditions, and they performed a series of elasto-plastic FEM analysis to examine the influence of lateral pressure on the buckling/plastic collapse and strength for the continuously stiffened plates. As a result of their analysis, they concluded:

- The buckling strength increases with the increase of applied lateral pressure.
- With the increase of the applied lateral pressure, the boundary condition of the panel between stiffeners changes from simply supported condition to clamped condition. This change increases the buckling/plastic collapse strength of stiffened plates.
- With larger lateral pressure, yielding starts earlier. This reduces the buckling/plastic collapse strength of stiffened plates.
- The formulae by classification societies give conservative buckling strength under bi-axial compression, and the bottom plating has much reserve strength when the lateral pressure is acting on it.

Since the presence of lateral pressure in most cases increases the plate strength, the effect of lateral pressure is therefore not included in the criteria in Eqn.(4).

2.2.2.2 Comparison Study – Ultimate Strength

Test Database

The plate test database consists of 206 datasets of test results for plates under uniaxial compression along their short edges and 15 tests for plates under uniaxial compression along their long edges. The data was collected from numerous publications (Frieze, 2002). Unfortunately, no test data in the database is related to plates under edge shear loading.

FEA Simulation

A numerical analysis of the ultimate strength for the test plates was executed based on a general nonlinear finite element program (ANSYS, 2004). The finite element analysis uses the full Newton-Raphson equilibrium iteration scheme and arc-length method to include geometric and material nonlinearities and to pass through the extreme points. The bisection and automatic time stepping features are activated to enhance the
convergence. The material is idealized to be elastic-perfectly-plastic. The finite strain “Shell 181” element type is used to model the plates. This element type is appropriate for linear, large rotation and/or large strain nonlinear applications and supports both full and reduced integration schemes. Simply supported boundary conditions are applied, as shown in Figure 9.

\[
W_p = \left[ \delta_{01} \sin\left( \frac{\pi x}{l} \right) + \delta_{03} \sin\left( \frac{\alpha \pi x}{l} \right) \right] \sin\left( \frac{\pi y}{s} \right) 
\]

where

\[
\delta_{01} = \frac{2W_p}{3l} \quad \text{and} \quad \delta_{03} = \frac{W_p}{3l} 
\]

The first term represents the dominant initial imperfection mode and the second term is the buckling mode of long plates with the aspect ratio of 3 (Davidson et al, 1989). The amplitude of the imperfection is selected based on statistics from ship plating by Smith, et al. (1987).

Two kinds of in-plane boundary conditions are used:

FEA1: All edges move freely during loading. This boundary condition is applicable in the single plate test.

FEA2: All edges remain straight during loading. The element types, “Beam4” and “Combin7”, are used to simulate the in-plane boundary condition. The special geometrical property with very small sectional area and very large moment of inertia is assigned to the beam element. This boundary condition is accepted in continuous plated structures as long as the stiffeners are strong enough so that they do not fail prior to buckling of plate.

Comparison Results

- Long Plates

Figure 10 shows the ultimate strength, \(\sigma_{ul}\), versus slenderness ratio, \(\beta\), in which the predictions from the ABS Buckling Guide (ABS, 2004a), DnV CN30.1 (DnV, 1995) and API BULLETIN 2V (API, 2000), test results and FEA simulation with freely movable in-plane edges (FEA1) are compared. The modeling uncertainty is shown in Figure 11. The mean and COV are given in Table 2. For long plate under axial compression, the ultimate strength criteria in ABS Buckling Guide (ABS, 2004a) is the same as in API BULLETIN 2V (API, 2000).

<table>
<thead>
<tr>
<th></th>
<th>API Bulletin 2V &amp; ABS Buckling Guide</th>
<th>DnV CN30.1</th>
<th>FEM1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.9559</td>
<td>1.0744</td>
<td>1.0114</td>
</tr>
<tr>
<td>COV</td>
<td>14.81%</td>
<td>14.95%</td>
<td>15.98%</td>
</tr>
</tbody>
</table>

FEA1 provides the best match to the test data. The mean of the modeling uncertainty from API Bulletin 2V (API, 2000) and the ABS Buckling Guide (ABS, 2004a) is less than one, whereas the mean of the modeling uncertainty from FEA1 and DnV CN30.1 (DnV, 1995) is slightly greater than one. This implies that almost all test plates are for the boundary condition freely moving in-plane during loading. As indicated above, this type of boundary conditions may possibly be suitable for single plates, but they can not represent the real conditions of plate panels in continuous plate structure. Although the formula from DnV (DnV, 1995) seems to underestimate the ultimate strength of plates in the continuously plated structures, DnV MOU (DnV, 1996) used a higher adjustment coefficient of 1.1 to modify the estimate. After this coefficient applies, the mean of modeling uncertainty based on DNV CN30.1 (DnV, 1995) reduces to 0.9767.

Figure 12 shows a comparison of the ultimate strength, \(\sigma_{ul}\), versus slenderness ratio, \(\beta\), in which the boundary condition that retains straight edges during loading is applied for the FEA simulation, i.e., FEA2. The modeling uncertainty of the prediction based on FEA2 is shown in Figure 13. The mean and COV are given in Table 3.

<table>
<thead>
<tr>
<th></th>
<th>API Bulletin 2V &amp; ABS Buckling Guide</th>
<th>DnV CN 30.1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>1.0111</td>
<td>1.1433</td>
</tr>
<tr>
<td>COV</td>
<td>4.17%</td>
<td>11.34%</td>
</tr>
</tbody>
</table>
Figure 10: Ultimate Strength of Long Plates

Figure 11: Modeling Uncertainty of Long Plates

Figure 12: Ultimate Strength of Long Plates

Figure 13 Modeling Uncertainty of Long Plates

Wide Plates

Figure 14 shows a comparison of the ultimate strength, $\sigma_{Ux}/\sigma_0$, versus slenderness ratio, $\beta$, based on the formulae of DnV CN30.1 (DnV, 1995), API BULLETIN 2V (API, 2000) and the ABS Buckling Guide (ABS, 2004a), test results and FEA simulation with the boundary conditions that remain straight edges during loading (FEA2). The mean and COV of modeling uncertainty with the base of FEA2 are given in Table 4.
Figure 14: Ultimate Strength of Wide Plates

Table 4: Mean/COV of the Modeling Uncertainty of FEM2/Predictions

<table>
<thead>
<tr>
<th></th>
<th>DnV CN30.1</th>
<th>API Bulletin 2V</th>
<th>ABS Buckling Guide</th>
</tr>
</thead>
<tbody>
<tr>
<td>MEAN</td>
<td>1.0038</td>
<td>1.1250</td>
<td>0.9852</td>
</tr>
<tr>
<td>COV</td>
<td>11.95%</td>
<td>13.33%</td>
<td>6.62%</td>
</tr>
</tbody>
</table>

- **Edge Shear Loading**

Figures 15 provides the comparison of the ultimate strength, $\frac{\tau_U}{\tau_0}$, versus slenderness ratio, $\beta$, based on the formulae of DnV CN30.1 (DnV, 1995), API BULLETIN 2V (API, 2000) and the ABS Buckling Guide (ABS, 2004a) and FEA simulation (FEA2) with the different aspect ratio, slenderness ratio and edge shear loading. The results presented here are based on the strain control condition applied along the panel edges, which represents the presence of stiffeners and flanges. A limiting shear yield strain has been set when evaluating the ultimate strength from a design point of view in order to limit the overall panel shear deformation.

- **Combined In-plane Biaxial Compression and Shear**

A large number of comparisons have been performed of the interaction equations for buckling stress limit and ultimate strength among DnV CN30.1 (DnV, 1995), API Bulletin 2V (API, 2000), the ABS Buckling Guide (ABS, 2004a) and FEA simulation (FEA2) with the different aspect ratio, slenderness ratio and edge shear loading. Figure 16 and Figure 17 display some typical comparison results.
Based on the comparison of the results, it is concluded that the recommended criterion in the ABS Buckling Guide predicts the reasonable capacities and the few cases with differences from the nonlinear FEA are acceptable as compared with the existing offshore codes.
2.2.3 Uniform lateral pressure

2.2.3.1 Criteria

A plate panel subject to uniform lateral pressure alone or combined with in-plane stresses is required to satisfy the following equation:

\[ q_u \leq \eta 4.0 \sigma_0 \left( \frac{t}{s} \right) \left( 1 + \frac{1}{\alpha^2} \right) \sqrt{1 - \left( \frac{\sigma_{\text{eq}}}{\sigma_0} \right)^2} \]  

(6)

Since this equation is based on the plastic yield line method, the plate subject to combined lateral pressure and in-plane stresses should also fulfill the requirements as specified in Section 2.2.2.

2.2.3.2 Comparison Study

Figure 17 shows the effect of the permanent set without in-plane stresses applied. Increasing the amount of permissible permanent set always increases the ultimate strength of plates under lateral pressure. Figure 18 shows ultimate strength as a function of the ratio of thickness to width without in-plane stress applied. The API formula is overly conservative for the plate subjected to lateral pressure alone.

The Monte Carlo Simulation Technique is used to compare the ABS Buckling Guide (ABS, 2000) formula with the DnV CN30.1 (DnV, 1995) formula for the plates subjected to combined lateral pressure and in-plane stresses. The range of the basic variables covered by the sampling points is given in Table 5. The 5000 simulations were carried out. The sampling sets were established from uniform random number generator. If the equivalent stress obtained is greater than the yield stress, then the sampling set is discarded. The comparative results are shown in Figure 19. The ratio mean and coefficient of variation (COV) of of the ABS Buckling Guide (2004) formula to the DnV formula, excluding the adjustment factor of 1.1, are 1.08 and 0.08 respectively. In general, the predictions from the ABS Buckling Guide formula are in quite close agreement with those from the DnV CN30.1 (DnV, 1995) formula.

2.3 Stiffened Panels

2.3.1 Beam column buckling state limit

The beam-column buckling state limit may be determined as follows:

\[ \frac{\sigma_0}{\eta \sigma_{CA}(A_e/A)} + \frac{C_m \sigma_b}{\eta \sigma_0 [1 - \sigma_0/(\eta \sigma_{E(C)})]} \leq 1 \]  

(7)
where,
\[ \sigma_{CA} = \sigma_{E(C)} \quad \text{for } \sigma_{E(C)} \leq P_t \sigma_0 \]
\[ = \sigma_0 [1 - P_r (1 - P_t)] \frac{\sigma_0}{\sigma_{E(C)}} \quad \text{for } \sigma_{E(C)} > P_t \sigma_0 \]
\[ \sigma_{E(C)} = \frac{\pi^2 E r_e^2}{l^2} \]
\[ A = A_s + st, \]
\[ A_e = A_s + s_e l \]
\[ s_e = s, \text{ when the buckling state limit of the associated plating from Eqn. (1) is satisfied} \]
\[ = C_x C_y C_{xy} s, \text{ when the buckling state limit of the associated plating from Eqn. (1) is not satisfied} \]
\[ C_y = 0.5 \varphi \left( \frac{\sigma_{y_{\max}}}{\sigma_{y}} \right) + \sqrt{1 - (1 - 0.25 \varphi^2) \left( \frac{\sigma_{y_{\max}}}{\sigma_{y}} \right)^2} \]
\[ C_{xy} = \sqrt{1 - \left( \frac{r}{r_0} \right)^2} \]
\[ r_e = \sqrt{\frac{I_e}{A_e}} \]

\( I \) is the unsupported span of longitudinal as defined in Figure 20.

\( SM_w \) is the effective section modulus of the longitudinal at flange, accounting for the effective breadth, \( s_w \), as specified in Figure 21. A limit for \( C_y \) is that the transverse loading should be less than the transverse ultimate strength of the plate panels. The buckling check for stiffeners is not to be performed until the attached plate panels satisfy the ultimate strength criteria.

### 2.3.2 Torsional-flexural buckling state limit

#### 2.3.2.1 Criteria

In general, the torsional-flexural buckling state limit of stiffeners or longitudinals is to satisfy the ultimate state limit given below:
\[ \frac{\sigma_a}{\eta \sigma_{CT}^x} \leq 1 \quad (8) \]

\[ \sigma_{CT} = \begin{cases} \sigma_{C_T} & \text{if } \sigma_{E_T} \leq P_t \sigma_0 \\ \sigma_0 \left[ 1 - P_r (1 - P_t) \right] \frac{\sigma_0}{\sigma_{E_T}} & \text{if } \sigma_{E_T} > P_t \sigma_0 \end{cases} \]
\[ \sigma_{E_T} = \frac{K}{2.6} \left( \frac{q \pi}{l} \right)^2 \left( \frac{I_o + C_o \left( \frac{l}{q \pi} \right)^2}{E \sigma_{CT}^2} \right) \]
\[ K = b l_f^3 + d_w l_w^3 \]
\[ I_o = I_y + m l_z + A_s \left( y_o^2 + z_o^2 \right) \]

---

**Figure 20: Unsupported span of longitudinal**

**Figure 21: Effective Breadth of Plating \( s_w \)**
\[ m = 1.0 - u \left( 0.7 - 0.1 \frac{d_w}{b_f} \right) \]

\[ u = 1 - 2 \frac{b_1}{b_f}, \text{ unsymmetrical factor} \]

\[ C_o = \frac{E t^3}{3s} \]

\[ \Gamma \cong m I_c d_w^2 + \frac{d_w^3 t_w^2}{36} \]

\[ I_{zf} = \frac{t f b_2^2}{12} \left( 1 + 3.0 \frac{u^2 d_u t_w}{A_s} \right) \]

\[ \sigma_{cL} = \frac{\pi^2 E \left( \frac{p + \alpha}{p} \right)^2 \left( \frac{t}{s} \right)}{12 \left( 1 - \nu^2 \right)} \]

### 2.3.2.2 Comparison Study

#### Test Database

The database consists of 359 test datasets of stiffened plates subjected to uniaxial in-plane compression and lateral pressure, see Frieze (2002). The compression flanges of the box girders have also been included because of their close similarities to deck structures. A detailed set of assessments has been conducted using the formulations of API Bulletin 2V (API, 2000), DnV CN30.1 (DnV, 1995) and the ABS Buckling Guide (ABS, 2004a).

Table 6 provides the mean and COV of modeling uncertainty of API Bulletin 2V (API, 2000), DnV CN30.1 (DnV, 1995) and the ABS Buckling Guide (ABS, 2004a) based on the test database before screening. Figure 22 shows the distribution of modeling uncertainty after screening. The screening condition is that the stiffeners satisfy the stiffness requirement specified in Section 3/9.1 of the ABS Buckling Guide (ABS, 2004a).

For all data available, the ABS Buckling Guide (ABS, 2004a) provides good predictions compared to the test database.

### 2.4 Local buckling of web, flange and face plate

The local buckling of stiffeners is to be assessed if the proportions of stiffeners specified in Section 3/9 in the ABS Buckling Guide (ABS, 2004a) are not satisfied.

- **For the web plate**

Critical buckling stress can be obtained from Section 2.2 by replacing \( s \) by the web depth and \( l \) by the unsupported span,

\[ \sigma_{EW} = k_s \frac{\pi^2 E}{12\left( 1 - \nu^2 \right)} \left( \frac{t_w}{d_w} \right)^2 \]  \hspace{1cm} (9)

\[ k_s = 4C_s \]

\[ C_s = 1.0, \text{ for angle or tee bar} \]

\[ = 0.33, \text{ for bulb plates} \]

\[ = 0.11, \text{ for flat bar} \]

- **For the flange and face plate**

Critical buckling stress can be obtained from Section 2.2 by replacing \( s \) by the larger outstanding dimension of flange, \( b_2 \) (Figure 2), and \( l \) by the unsupported span,

\[ \sigma_{EF} = 0.44 \frac{\pi^2 E}{12\left( 1 - \nu^2 \right)} \left( \frac{t_f}{b_2} \right)^2 \]  \hspace{1cm} (10)

### 3 ANALYSIS OF HULL GIRDER ULTIMATE STRENGTH

#### 3.1 General

The hull girder ultimate strength, \( M_u \), is defined as the maximum bending capacity of the hull girder beyond which the hull will collapse. Hull girder failure is controlled by the ultimate strength of structural elements, considering buckling and yielding. The hull girder ultimate strength state limit may be defined by (Wang, Jiao and Moan, 1996; Sun and Bai, 2001 and Sun and Guedes Soares, 2003):

\[ \gamma_s M_s + \gamma_w \phi_w M_w \leq \frac{M_u}{\gamma_u} \]  \hspace{1cm} (11)

The characteristic hull girder ultimate strength of a hull girder section, in sagging or hogging, \( M_{sag} \) or \( M_{hog} \), is defined as the maximum value on the quasi static nonlinear bending moment-curvature relationship, \( M-\chi \), see Figure 23. The curve represents the progressive behavior of hull girder collapse under vertical bending. When the absolute value of curvature, \( \chi \), increases due to an externally applied hogging or sagging bending moment, the internal resistance, \( M \), of the hull cross...
section also increases, up to a point where, \(dM/d\chi\) becomes zero, or changes sign, which defines the ultimate longitudinal bending strength \(M_{ul}\) or \(M_{us}\).

Hull girder ultimate capacity can be, in general, calculated by nonlinear finite element analysis. Alternatively, it can also be obtained based on an incremental-iterative approach, as described in Section 3.3.

### Table 6: Mean/COV of Modeling Uncertainty for Stiffened Panels

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<th>Data Sources</th>
<th>Number</th>
<th>API Bulletin 2V MEAN</th>
<th>API Bulletin 2V COV</th>
<th>DnV CN30.1 MEAN</th>
<th>DnV CN30.1 COV</th>
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</tr>
</tbody>
</table>

3.2 Assumption

The method for calculating the ultimate hull girder capacity is to identify the critical failure modes of the main longitudinal structural elements. For example, for ships, in sagging, the critical mode is generally inter-frame buckling, as illustrated in Figure 24.

Figure 24: Ships in extreme sagging, inter-frame buckling failure

Structural elements compressed beyond their buckling limit have reduced strength according to their buckling and ultimate strength characteristics. All relevant failure modes for individual structural elements, such as plate...
buckling, beam-column buckling, torsional stiffener buckling, local buckling of stiffener, and their interactions, are to be considered in order to identify the weakest inter-frame failure mode.

For ship type structures, vertical bending is the dominant load; therefore, ultimate strength is based on resisting vertical bending from still water and wave-induced loadings.

In applying the incremental-iterative approach, the following assumptions and limits are generally to be observed:

- The ultimate strength is calculated for the hull transverse sections between two adjacent transverse frames;
- The transverse main supporting members, including frames, girders and floors, with their associated effective plating are to have a moment of inertia not less than \( I_G \) obtained from the following equation:
  \[
  I_G / I_o \geq 0.2(B/l)^{3/2}(B/s)
  \]
  - The hull transverse section remains plane during each curvature increment application;
  - The hull material has an elasto-plastic behavior;
  - The element stress, \( \sigma \), corresponding to the element strain, \( \varepsilon \), is selected as the minimum stress among the values obtained from each of the relevant load-end shortening curve, \( \sigma - \varepsilon \), in Section 3.4;
  - The hull transverse section is divided into a set of individual elements; the elements while considered to be acting independently are combined to provide the ultimate strength resistance of the hull’s transverse cross section. These elements are:
    - Plate element, for unstiffened plates;
    - Stiffener element, consisting of a stiffener with an associated effective width of plating;
    - Corner element, consisting of a plate intersection with a web plate.

The individual elements are illustrated in Figure 25.

### 3.3 Calculation of hull girder ultimate capacity

In this section, the procedure for calculating hull girder ultimate capacity based on the incremental-iterative approach is described.

Each step of the incremental procedure is represented by the calculation of the bending moment \( M' \), which acts on the hull transverse section as the effect of an imposed curvature \( \chi' \).

For each step, the value \( \chi' \) is obtained by adding an increment of curvature, \( \Delta \chi \), to the curvature \( \chi' \) value from the previous step. The increment of curvature corresponds to an increment of rotation of the hull’s transverse section around its instantaneous horizontal neutral axis.

![Figure 25: Example of individual structure elements](image)

Girder with web stiffeners  Girder without web stiffeners

Figure 25: Example of individual structure elements

The rotation angle increment induces axial strain, \( \varepsilon \), in each structural element of the hull section, whose value depends on the distance between the element’s location and the instantaneous horizontal neutral axis. In the sagging condition, the structural elements above the instantaneous horizontal neutral axis are shortened, whereas the elements below the instantaneous horizontal neutral axis are lengthened. This is reversed in the hogging condition.

The structural element stress, \( \sigma \), induced by strain, \( \varepsilon \), is to be obtained from the load-end shortening curve \( \sigma - \varepsilon \) of the element, as described in Section 3.4, which takes into account the nonlinear elasto-plastic behavior of the element. The stress in each element is converted to a force. The equilibrium of the element forces is then used in an iterative process to determine the instantaneous horizontal neutral axis position of the hull’s transverse cross-section.

Once the position of the instantaneous horizontal neutral axis is determined with the relevant element force distribution, the bending moment capacity of the section, \( M' \), about the instantaneous horizontal neutral axis, is obtained by combining the contribution from each element.

Figure 26 is a flow chart showing the main steps of the incremental-iterative approach.

### 3.4 Load-end shortening curve

The nonlinear material behavior for in-plane tension or compression is different for different element types. When a structural element is in tension, full plasticity beyond yield (up to a rupture limit) is normally anticipated. However, when a structural element is under compression, elasto-plastic material and nonlinear geometric behavior appear. The tensile and compressive behavior can be described by the so-called, ‘load-end shortening’ curves, as described below.
3.4.1 Plate element

Unstiffened plates comprising the hull transverse sections may collapse in one of two failure modes:

- Yielding in tension;
- Buckling in compression.

\[
\chi^i = \min_{i=1,\ldots, N} \left\{ \frac{\sigma_{C,i} z_i + \sigma_{U,i}}{z_i^2} \right\} / E
\]

where \( \sigma_{C,i} \) and \( \sigma_{U,i} \) are respectively the critical buckling strength and specified minimum yield point of the \( i \)th structural element, respectively; \( E \) is the elastic modulus; \( z_i \) is the distance of the \( i \)th structural element to the neutral axis and \( N \) is the total number of structural elements.

Figure 26: Flow chart of the procedure for the evaluation of the bending moment - curvature curve, \( M-\chi \)
Yielding in tension

When an unstiffened plate is stretched in tension, the load-end shortening curve, $\sigma - \varepsilon$, is idealized as the elastic-perfectly plastic relationship, as explained below and shown in Figure 27.

$$\frac{\sigma}{\sigma_0} = \begin{cases} \varepsilon & \text{for } 0 \leq \varepsilon \leq 1 \\ 1 & \text{for } \varepsilon > 1 \end{cases} \quad (12)$$

Buckling in compression

The stress acting on an unstiffened plate, $\sigma_{UP}^E$, should be limited to its ultimate strength, $\sigma_{UX}$, and not less than critical buckling stress, $\sigma_{CP}$, as specified in the following. The load-end shortening curve, $\sigma - \varepsilon$, for unstiffened plate buckling, is shown in Figure 27 and is defined by the following equation:

When $\varepsilon \leq \sigma_{UX}/\sigma_0$

$$\sigma_{UP}^E = \sigma_0 \varepsilon \quad (13-1)$$

When $\varepsilon > \sigma_{UX}/\sigma_0$

$$\frac{\sigma_{UP}^E}{\sigma_0} = \begin{cases} C \frac{\varepsilon}{\bar{\varepsilon}} & \text{for } \alpha \geq 1 \\ C \frac{\varepsilon}{\bar{\varepsilon}} + 0.1 \left[ 1 - \frac{s}{l} \right] \left[ 1 + \frac{1}{\beta_2^2} \right] & \text{for } \alpha < 1 \end{cases}$$

$$\sigma_{CP} \leq \sigma_{UP}^E \leq \sigma_{UX} \quad (13-2)$$

where

$$C_E = 1.0 \text{ for } \beta_E \leq 1$$

$$= 2/ \beta_E - 1/ \beta_E^2 \text{ for } \beta_E > 1$$

$$\beta_E = s/l \sqrt{E_n} \sigma_{0p}/E$$

$$\sigma_{CP}^E = \frac{\sigma_{Ex}}{E_n} \text{ for } \sigma_{Ex} \leq P_s \sigma_0 \varepsilon_n$$

$$= \sigma_0 \left[ 1 - P_s \left( 1 - P_s \right) \frac{\sigma_{0p}}{\sigma_{Ex}} \right] \text{ for } \sigma_{Ex} > P_s \sigma_0 \varepsilon_n$$

3.4.2 Stiffener element

A longitudinal plate stiffener (i.e., axis is normal to the hull’s transverse section) may fail in one of four modes:

- Yielding in tension;
- Beam-column buckling;
- Torsional-flexural buckling;
- Local buckling of stiffeners.

The load-end shortening curves, $\sigma - \varepsilon$, for each failure mode is described below.

**Yielding in tension**

The load-end shortening curve for yielding in tension is the same as in Section 3.4.1.

**Beam-column buckling**

The load-end shortening curve, $\sigma - \varepsilon$, for beam-column buckling is shown in Figure 28 and defined by the following equation:

When $\varepsilon \leq \sigma_{CA}/\sigma_0$

$$\sigma_{CI} = \sigma_0 \varepsilon \quad (14-1)$$

When $\varepsilon > \sigma_{CA}/\sigma_0$

$$\sigma_{CI} = \sigma_{CA}^E \frac{A_s + s_E}{A_s} \leq \sigma_{CA} \frac{A_s + s_l}{A_s} \quad (14-2)$$

where

$$\sigma_{CA}^E = \frac{\sigma_{E(C)}^n}{E_n} \text{ for } \sigma_{E(C)} \leq P_s \sigma_0 \varepsilon_n$$

$$\sigma_{CA}^E = \sigma_0 \left[ 1 - P_s \left( 1 - P_s \right) \frac{\sigma_{0p}}{\sigma_{Ex}} \right] \text{ for } \sigma_{E(C)} > P_s \sigma_0 \varepsilon_n$$

$$s_E^C = C_E s$$

$$C_E = 1.0 \text{ for } \beta_E \leq 1.0$$

$$= 2 \beta_E - 1/ \beta_E^2 \text{ for } \beta_E > 1.0$$

Figure 27: Load-end shortening curve for plate ultimate strength

Figure 28: Load-end shortening curve for beam-column buckling
Torsional-flexural buckling

The load-end shortening curves, $\sigma - \varepsilon$, for torsional-flexural buckling is presented in Figure 29 and defined by the following equation:

When $\varepsilon \leq \sigma_{\text{CT}} / \sigma_0$

$$\sigma_{C2} = \sigma_0 \frac{E}{\varepsilon}$$

(15-1)

When $\varepsilon > \sigma_{\text{CT}} / \sigma_0$

$$\sigma_{C2} = \frac{\sigma_{\text{CT}}^E A_\varepsilon + \sigma_{UP}^E}{A_\varepsilon + st}$$

(15-2)

where

$$\sigma_{\text{CT}}^E = \frac{\sigma_{\text{ET}}}{\varepsilon^n}$$

for $\sigma_{\text{ET}} \leq P_r \sigma_0 \varepsilon^n$

$$= \sigma_0 [1 - P_r (1 - P_r) \frac{\sigma_0 \varepsilon^n}{\sigma_{\text{ET}}}]$$

for $\sigma_{\text{ET}} > P_r \sigma_0 \varepsilon^n$

$\sigma_{\text{CT}}^E$ should be less than $\sigma_{\text{CT}}$

![Figure 29: Load-end shortening curve for torsional – flexural buckling](#)

Local buckling of stiffeners

This failure mode should be assessed if the proportions of stiffeners specified in Section 3/9 in the ABS Buckling Guide (ABS, 2004a) are not satisfied.

The load-end shortening curve, $\sigma - \varepsilon$, for local buckling of stiffeners is shown in Figure 30 and defined by the following equation:

When $\varepsilon \leq \sigma_{\text{CL}} / \sigma_0$

$$\sigma_{C3} = \sigma_0 \frac{E}{\varepsilon}$$

(16-1)

When $\varepsilon > \sigma_{\text{CL}} / \sigma_0$

$$\sigma_{C3} = \frac{\sigma_{\text{CL}}^E A_\varepsilon + \sigma_{UP}^E}{A_\varepsilon + st}$$

(16-2)

where

$$\sigma_{\text{CL}}^E = \frac{\sigma_{\text{EL}}}{\varepsilon^n}$$

for $\sigma_{\text{EL}} \leq P_r \sigma_0 \varepsilon^n$

$$= \sigma_0 [1 - P_r (1 - P_r) \frac{\sigma_0 \varepsilon^n}{\sigma_{\text{EL}}}]$$

for $\sigma_{\text{EL}} > P_r \sigma_0 \varepsilon^n$

$\sigma_{\text{CL}}^E$ should be less than $\sigma_{\text{CL}}$

![Figure 30: Load-end shortening curve for local buckling](#)

3.4.3 Corner element

Corner elements are considered stocky elements, which collapse due to ‘fully plastic’ development. The relevant load-end shortening curve, $\sigma - \varepsilon$, is idealized by the elastic-perfectly plastic relationship given next and shown in Figure 31:

$$\sigma = \begin{cases} -1 & \text{for } \varepsilon < -1 \\ \varepsilon & \text{for } -1 \leq \varepsilon \leq 1 \\ 1 & \text{for } \varepsilon > 1 \end{cases}$$

![Figure 31: Load-end shortening curve for a corner element](#)

3.4.4 Post buckling analysis of stiffened panel

To identify the post-buckling behaviors of stiffened panels, e.g., exponential value, $n$, the series of nonlinear analyses are performed using a commercial FEA code (ANSYS, 2004) program. The engineering model to be used in the finite element analysis is shown in Figure 32 (Mansour, 2003). The material is idealized to be elasto-perfectly-plastic and the fabrication-induced residual stresses are ignored. The initial deflection mode is taken as the first buckling mode as shown in Figure 33, which
deteriorates the ultimate strength most significantly. Three levels of initial deflection amplitude are taken into account in order to demonstrate the conservatism degree of stiffened panel state limit, which are:

\[ W_{0L} = 0.10\%d, 0.15\%d \text{ and } 0.20\%d \]

Where \( W_{0L} \) is the initial deflection amplitude.

### 4.1 Single Hull VLCC—Energy Concentration

The hull girder of a 10-year-old Very Large Crude Carrier (VLCC), Energy Concentration, broke while unloading in 1980. This accident was reported in detail by Rutherford and Caldwell in their paper (1990).

Because a significant percentage of FPSOs have been converted from the single hull tankers in order to reduce the construction costs, the example can be used to validate the ultimate longitudinal bending strength predictions for converted FPSOs.

### 4 CASE STUDY

Hull girder ultimate strength analysis for different types of FPSOs, including single hull, double hull and single bottom – double sides, is carried out. The predictions from the proposed procedure are comparable with those presented by Yao, et al (1992). The two examples, as included in the ISSC 2000 Special Task Committee V1.2, by Yao, et al (2000), are demonstrated in this paper in order to verify the accuracy and efficiency of the procedure.

![Figure 32: Engineering Model](image)

![Figure 33: Initial Imperfection Mode](image)

The type of element, finite strain Shell 181, is used to model the plate, as explained in Section 2.2.2.2. The element is well suited for linear, large rotation and/or large strain nonlinear applications and supports both full and reduced integration schemes.

Figure 34 shows some representative results among others based on the nonlinear FE analysis together with the predictions from the proposed formula in Section 3.4.

It can be seen that the ultimate strength predictions by the formula for all types of stiffened panels is in quite good agreement with the results by FEA. The effect of initial imperfection is significant on the ultimate strength of stiffened panels, but is relatively small on the postbuckling behaviors. It is therefore recommended that the exponential value, \( n \), is set at 2 in the proposed formula in order to keep an acceptable conservative degree in postbuckling range of stiffened panels.

![Figure 34: Load – end shortening curves by nonlinear FEA against the prediction from the proposed formulae](image)
The midship section of the vessel is reproduced in Figure 35. In the method presented in this paper, the cross section is modeled using two element types, i.e., stiffener elements and corner elements. A total of 104 stiffener elements and 18 corner elements. The calculated results of hull girder ultimate strength are summarized in Table 8. The moment-curvature response based on the present procedure is provided in Figure 36.

Figure 35: Midship Section of the Single Hull VLCC

The calculated results of hull girder ultimate strength are summarized in Table 7. The moment-curvature response based on the present procedure is provided in Figure 36.

Figure 36: Moment-Curvature Response of SH VLCC

4.2 Double Hull VLCC

Double hull structures are arranged in some new-built FPSOs although this is not required according to IMO Circular MEPC 406 (IMO, 2003). This structural pattern is very similar to the double hull tankers. In order to validate the ultimate longitudinal bending strength predictions for the double hull structures, the example taken from ISSC 2000 Special Task Committee V1.2 Report is provided in this subsection.

Figure 37 shows the midship section of the vessel. The cross section is modeled using 198 stiffener elements and 18 corner elements. The calculated results of hull girder ultimate strength are summarized in Table 8. The moment-curvature response based on the proposed procedure is provided in Figure 38.

Figure 37: Midship Section of DH VLCC

Figure 38: Moment-Curvature Response of DH VLCC

5 SUMMARY AND CONCLUSIONS

In the buckling and ultimate strength assessment for FPSO structures, the potential failure modes for plates and stiffened panels under combined loads have been considered, which include:

- Buckling and collapse of plates - Lateral deflection develops in post-buckling region and ultimate strength is reached due to elasto-plastic collapse
Table 7: Ultimate Hull Girder Strength of Single Hull VLCC*, **

<table>
<thead>
<tr>
<th>Items</th>
<th>Astrup</th>
<th>Chen</th>
<th>Cho</th>
<th>Dow</th>
<th>Masaoka</th>
<th>Rigo(1)</th>
<th>Rigo(2)</th>
<th>Yao</th>
<th>ABS</th>
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<tbody>
<tr>
<td>$I_x$ (m$^4$)</td>
<td>819.7</td>
<td>743.8</td>
<td>828.3</td>
<td>812.86</td>
<td>800.89</td>
<td>816.47</td>
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<td>$z_{Gy}$ (m)</td>
<td>12.15</td>
<td>11.85</td>
<td>12.01</td>
<td>12.81</td>
<td>12.27</td>
<td>12.22</td>
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<td></td>
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<tr>
<td>$M_{p}$ (x10$^6$MN-m)</td>
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<td>19.85</td>
<td>21.96</td>
<td>12.31</td>
<td>20.39</td>
<td>21.75</td>
<td>21.46</td>
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</tr>
<tr>
<td>$M_{V_{y}}$ (x10$^6$MN-m)</td>
<td>19.7</td>
<td>17.47</td>
<td>19.69</td>
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<td>20.57</td>
<td>19.43</td>
<td>19.84</td>
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<tr>
<td>$M_{V_{y}}$ (x10 MN-m)</td>
<td>21.16</td>
<td>19.68</td>
<td>21.63</td>
<td>19.9</td>
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<td>21.02</td>
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<tr>
<td>$M_{d}$ (x10$^6$MN-m)</td>
<td>17.29</td>
<td>16.08</td>
<td>17.67</td>
<td>16.81</td>
<td>15.87</td>
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<tr>
<td>$M_{iul}$ (x10$^6$MN-m)</td>
<td>17.16</td>
<td>18.54</td>
<td>16.75</td>
<td>16.26</td>
<td>16.73</td>
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<tr>
<td>$M_{u}$ (x10 MN-m)</td>
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<td>20.23</td>
<td>20.09</td>
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<td>18.46</td>
<td>17.54</td>
<td>18.22</td>
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</tbody>
</table>

Table 8: Ultimate Hull Girder Strength of Double Hull VLCC*

<table>
<thead>
<tr>
<th>Items</th>
<th>Chen</th>
<th>Cho</th>
<th>Masaoka</th>
<th>Rigo(1)</th>
<th>Rigo(2)</th>
<th>Soares**</th>
<th>Yao</th>
<th>ABS</th>
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<td>$M_{p}$ (x10$^6$MN-m)</td>
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<tr>
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<td>19.68</td>
<td>19.82</td>
<td>20.61</td>
<td>20.24</td>
<td>19.66</td>
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<tr>
<td>$M_{iul}$ (x10$^6$MN-m)</td>
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<td>27.17</td>
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<tr>
<td>$M_{u}$ (x10 MN-m)</td>
<td>24.33</td>
<td>20.80</td>
<td>26.59</td>
<td>19.62</td>
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<td>25.61</td>
<td>27.61</td>
<td>28.88</td>
<td>30.99</td>
</tr>
</tbody>
</table>

(*: All the results, except ABS, are taken from Yao, et al. ISSC 2000 Special Task Committee V1.2 Report “Ultimate Hull Girder Strength”. **: Estimated applied load at collapse in hogging is 17.94x10$^6$ MN-m from Rutherford and Caldwell (1990). ***: The values of $M_{US}$ and $M_{UB}$ by Guedes Soares correspond to Case (1) as indicated in the report).

Collapse of stiffeners

- Beam-column buckling in which attaching plates are accounted for as effective plating
- Tripping of stiffeners – Tripping due to buckling of stiffeners and loss of the rotational restraint provided by the plating
- Local buckling of stiffeners – Web and flange buckling due to inappropriate proportions of stiffeners
  - Grillage buckling
  - Hull girder ultimate strength

It is recommended that the FPSO structure is ideally designed in such a way that the buckling and ultimate strength of each level is greater than its preceding level, i.e., a well designed structure does not collapse when a plate fails as long as the stiffeners can resist the extra load they experience from the plate failure. Even if the stiffeners collapse, the structure may not fail immediately as long as the grillage can support the extra load shed from the stiffeners. Hull girder ultimate strength is the maximum capacity of the FPSO structure beyond which the catastrophic collapse occurs. Therefore, suitable scantling proportions between plates, stiffeners and transverse web/girders are necessary to guarantee the sufficient safety of the FPSO structure.

Because plate panels exhibit a continued increase of resistance after buckling, before they finally reach the ultimate load carrying capacity, it is acceptable that the plate panels are designed to reach the buckling state but not ultimate state. In this case, the reduced effective plating width should be taken into account in the assessment of stiffened panels.

This paper provided the buckling and ultimate strength assessment criteria and their fundamental principles and technical background for FPSO structures. The safety and accuracy for determining buckling and the ultimate strength predictions were established by the comparison of its results against a very extensive test database of failures collected by ABS and other data subjected to a variety of loading conditions. Results obtained using the criteria were also compared against existing recognized offshore standards and publications. The hull girder strength analysis procedure is verified by real damage data and other available methods.

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7 REFERENCES

ABS (2001), Rules for Building and Classing Offshore Mobile Drilling Units.


ABS (2004b), Rules for Building and Classing Steel Vessels.


ABS (2005b), Guidance Notes on Hull Girder Ultimate Strength Analysis for Ship-Type Offshore Installations. (to be published)

ANSYS (2004), Release 8.1 Documentation, ANSYS, INC.

API (2000), Design of Flat Plate Structures, API BULLETIN 2V.


DNV (1995), Buckling Strength analysis, DNV Classification Note 30.1.

DNV (1996), Rules for Classification of Mobile Offshore Units.


IMO (2003), Guidelines For Application of MARPOL Annex I Requirements To FPSOs And FSUs.


Simonsen, B C (2003), “Ultimate Strength”, ISSC Committee III.1 Report, 15th International Ship and Offshore Structures Congress, San Diego, USA.


